TOWNSHIP OF UNION PUBLIC SCHOOLS

UHS – Geometry

Curriculum Guide 2017

Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

Philosophy Statement

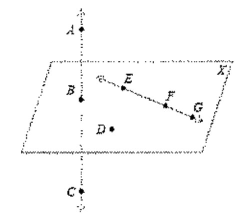
The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

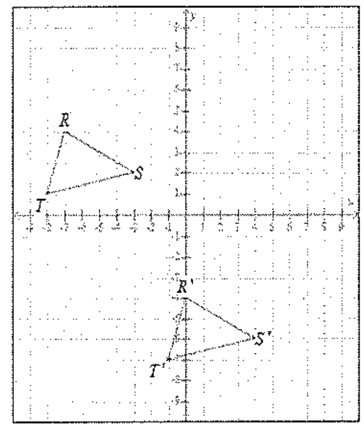
This course includes the topics: the language of geometry, reasoning and introduction to proof, parallels, congruent triangles and quadrilaterals. Scientific calculators will be used throughout the course. This course continues with the study of similar figures, right triangles, trigonometry, circles, probability and statistics, polygons and their areas, surface area and volume of prisms, pyramids, cylinders, cones, and spheres.

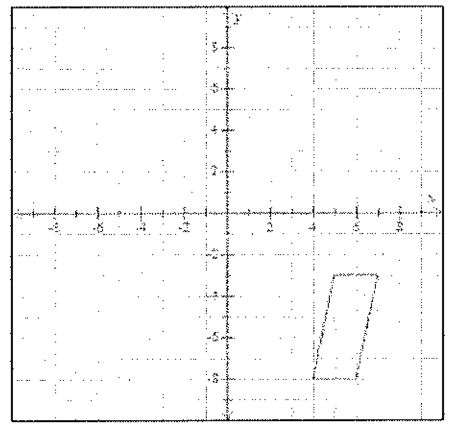
Overview	Standards for Mathematical Content		Unit Focus	Standards for Mathematical Practice
Unit 1 **Congruence and Constructions**	☐ G.CO.A.1 ☐ G.CO.A.2 ☐ G.CO.A.3 ☐ G.CO.A.4 ☐ G.CO.A.5	■ G.CO.B.6 ■ G.CO.B.7 ■ G.CO.B.8 ☐ G.CO.D.12 ☐ G.CO.D.13	• Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Make geometric constructions	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 1: Suggested Open Educational Resources	G.CO.A.1 Defining Parallel Lines G.CO.A.1 Defining Perpendicular Lines G.CO.A.2 Horizontal Stretch of the Plane G.CO.A.3 Seven Circles II G.CO.A.3 Symmetries of rectangles G.CO.A.4 Defining Rotations G.CO.A.5 Showing a triangle congruence		G.CO.B.7 Properties of Congruent Triangles G.CO.B.8 Why does SAS work? G.CO.B.8 Why does SSS work? G.CO.B.8 Why does ASA work? G.CO.D.12 Bisecting an angle G.CO.D.12 Angle bisection and midpoints of line segments G.CO.D.13 Inscribing an equilateral triangle in a circle	
Unit 2 **Congruence, Similarity & Proof**	■ G.SRT.A.1 ■ G.SRT.A.2 ■ G.SRT.A.3 ■ G.CO.C.9	■ G.CO.C.10 ■ G.CO.C.11 ■ G.SRT.B.4 ■ G.SRT.B.5	• Understand similarity in terms of similarity transformations • Prove geometric theorems. • Prove theorems involving similarity	
Unit 2: Suggested Open Educational Resources	G.SRT.A.1 Dilating a Line G.SRT.A.2 Are They Similar? G.SRT.A.2 Similar Triangles G.SRT.A.3 Similar Triangles G.CO.C.9 Congruent Angles made by parallel lines and a transverse G.CO.C.9 Points equidistant from two points in the plane		G.CO.C.10 Midpoints of Triangle Sides G.CO.C.10 Sum of angles in a triangle G.CO.C.11 Midpoints of the Sides of a Parallelogram G.CO.C.11 Is this a parallelogram? G.SRT.B.4 Joining two midpoints of sides of a triangle G.SRT.B.4 Pythagorean Theorem G.SRT.B.5 Tangent Line to Two Circles	

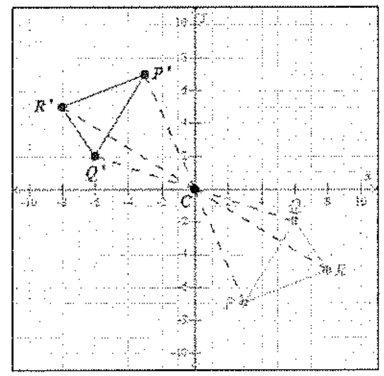
Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 3 **Trigonometric Ratios & Geometric Equations**	■ G.GPE.B.4 ■ G.GPE.B.5 ■ G.GPE.B.6 ■ G.GPE.B.7 ■ G.SRT.C.6 ■ G.SRT.C.7 ■ G.SRT.C.8 ◎ G.GPE.A.1 ◎ G.C.A.1 ◎ G.C.A.2 ◎ G.C.A.3 ◎ G.C.B.5	• Use coordinates to prove simple geometric theorems • Define trigonometric ratios and solve problems involving right triangles • Translate between the geometric description and the equation for a conic section • Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 3: Suggested Open Educational Resources	G.GPE.B.4,5 A Midpoint Miracle G.GPE.B.5 Slope Criterion for Perpendicular G.GPE.B.7 Triangle Perimeters G.SRT.C.6 Defining Trigonometric Ratio G.SRT.C.7 Sine and Cosine of Complimentary Angles	G.SRT.C.8 Constructing Special Angles G.GPE.A.1 Explaining the equation for a circle G.C.A.1 Similar circles G.C.A.2 Right triangles inscribed in circles I G.C.A.3 Circumscribed Triangles	
Unit 4 **Geometric Modeling**	■ G.MG.A.1 ◎ G.GMD.A.3 ◎ G.GMD.B.4 ■ G.MG.A.2 ■ G.MG.A.3 ◎ G.GMD.A.1	• Explain volume formulas and use them to solve problems. • Visualize relationships between two dimensional and three-dimensional objects • Apply geometric concepts in modeling situations	
Unit 4: Suggested Open Educational Resources	G.MG.A.1Toilet Roll G.GMD.A.3 The Great Egyptian Pyramids G.GMD.B.4 Tennis Balls in a Can G.MG.A.2 How many cells are in the human body? G.MG.A.3 Ice Cream Cone G.GMD.A.1 Area of a circle		

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
☐ G.CO.A.1. Know precise definitions of angle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, and distance along a line.	MP.6 Attend to precision.	Concept(s): • Point, line, plane, and distance along a line, as indefinable notions Students are able to: – use point, line, distance along a line – identify perpendicular lines (two lines are perpendicular if an angle formed by the two lines at the point of intersection is a right angle); – define parallel lines (distinct lines that have no point in common); – define line segment. Learning Goal 1: Use the undefined notion of a point, line, distance along a line to develop definitions for angles, parallel lines, perpendicular lines and line segments.	In the figure below, points B, D, E, F, and G lie in plane X. Points A and C do not lie in plane X. Check *all* statements that are true.  ☐ The lines $\overleftrightarrow{BC}$ and $\overleftrightarrow{CE}$ intersect. ☐ Another name for plane X is plane ABC. ☐ Points B, C, and D are collinear. ☐ Point D and the line $\overleftrightarrow{EF}$ are coplanar. ☐ Points A, E, and F are coplanar. ☐ None of these are true.

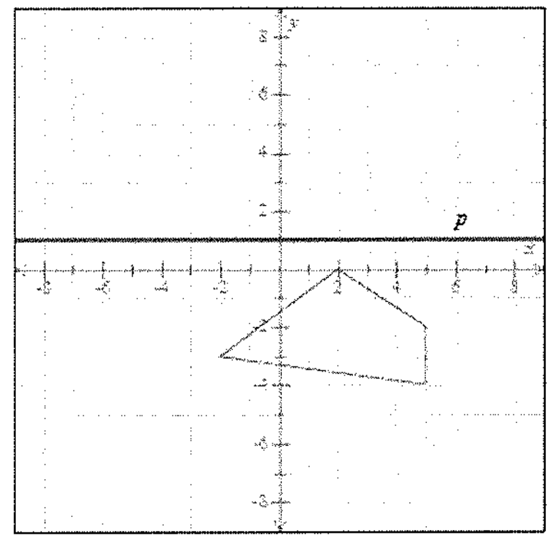
Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.GPE.B.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	MP.3 Construct viable arguments and critique the reasoning of others. MP.8 Look for and express regularity in repeated reasoning	Concept(s): No new concept(s) introduced Students are able to: • prove the slope criteria for parallel lines (parallel lines have equivalent slopes). • prove the slope criteria for perpendicular lines (the product of the slopes of perpendicular lines equals -1). • solve problems using the slope criteria for parallel and perpendicular lines. Learning Goal 2: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.	**Maps** Morris Avenue intersects both 1st Street and 3rd Street at right angles. 3rd Street is parallel to 5th Street. How are 1st Street and 5th Street related? Explain.

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
☐ G.CO.A.2. Represent transformations in the plane using, e.g., patty paper and geometry software; describe transformations as rules that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): • Transformations Students are able to: • represent transformations with geometry software (Geogebra) and patty paper. • describe rigid transformations as isometries (points defining the pre-image as the input and the points defining the image as the output). • describe a transformation by writing a rule • compare rotations, reflections, and translations to a horizontal stretch, vertical stretch and to dilations, distinguishing preserved distances and angles from those that are not preserved. Learning Goal 2: Represent transformations in the plane using geometric software, describe and explain transformations as rules, and compare rigid transformations to dilations, horizontal stretches and vertical stretches.	Triangle RST is translated 7 units to the right and 8 units down. The result is $\Delta R'S'T'$, as shown below.  (a) The arrows below show that the coordinates on the left are mapped to the coordinates on the right. Fill in the blanks to give the coordinates after the translation. original coordinates → final coordinates $R(-7,4) \rightarrow R'(\square,\square)$ $S(-3,2) \rightarrow S'(\square,\square)$ $T(-8,1) \rightarrow T'(\square,\square)$ (b) Choose the general rule below that describes the translation mapping ΔRST to $\Delta R'S'T'$. ○ $(x,y) \rightarrow (7x, -8y)$ ○ $(x,y) \rightarrow (x+7, y-8)$ ○ $(x,y) \rightarrow (x+8, y-7)$ ○ $(x,y) \rightarrow (8x, -7y)$ ○ $(x,y) \rightarrow (x-7, y+8)$ ○ $(x,y) \rightarrow (-7x, 8y)$ ○ $(x,y) \rightarrow (x-8, y+7)$ ○ $(x,y) \rightarrow (-8x, 7y)$

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
☐ G.CO.A.3. Given a rectangle, parallelogram, trapezoid, or regular polygons, describe the rotations and reflections that carry it onto itself.	MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to: • identify lines of symmetry when performing rotations and/or reflections on rectangles, parallelograms, trapezoids and regular polygons. • describe the rotations and reflections that carry rectangles, parallelograms, trapezoids and regular polygons onto itself. Learning Goal 3: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself, and identify lines of symmetry.	Reflect the figure across the y-axis. Then answer the questions below.  Are the statements below true or false? When the figure is reflected: If two sides are parallel to each other in the original figure, then those sides may *not* be parallel to each other in the final figure. ◌ True ◌ False When the figure is reflected, its angle measures stay the same. ◌ True ◌ False When the figure is reflected, the final side lengths are smaller than the original side lengths. ◌ True ◌ False

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
☐ G.CO.A.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	MP.6 Attend to precision.	Concept(s): • Impact of transformations on figures in the plane. Students are able to: • develop formal mathematical definitions of a rotation, reflection, and translation. Learning Goal 4: Develop formal definitions of rotations, reflections, and translations	Rotating ΔPQR by 180° clockwise about the origin, we get its image $\Delta P'Q'R'$ as shown below. Note that ΔPQR has vertices $P(3,-7)$, $Q(6,-2)$, and $R(8,-5)$. Also, note that $\Delta P'Q'R'$ has vertices $P'(-3,7)$, $Q'(-6,2)$, and $R'(-8,5)$. Complete the following.  (a) Suppose each pair of segments below have the same length. Find each length. Give exact answers (not decimal approximations). $CP = CP' =$ ☐ units $CQ = CQ' =$ ☐ units $CR = CR' =$ ☐ units (b) Suppose the angles below have the same measure. Choose the correct angle measure. Use the protractor provided, as necessary. $m\angle PCP' = m\angle QCQ' = m\angle RCR' =$? (c) Choose the correct pair of statements about the rotation. ○ Not every point on the original figure is the same distance from the center of rotation as its image. Not all angles formed by a point and its image, with the vertex at the center of rotation, are congruent. ○ Each point on the original figure is the same distance from the center of rotation as its image. Not all angles formed by a point and its image, with the vertex at the center of rotation, are congruent. ○ Not every point on the original figure is the same distance from the center of rotation as its image. All angles formed by a point and its image, with the vertex at the center of rotation, are congruent. ○ Each point on the original figure is the same distance from the center of rotation as its image. All angles formed by a point and its image, with the vertex at the center of rotation, are congruent.

<table>
<tr><th colspan="4">Unit 1 Geometry</th></tr>
<tr><th>Content Standards</th><th>Suggested Standards for Mathematical Practice</th><th>Critical Knowledge & Skills</th><th>Examples</th></tr>
<tr><td>☐ G.CO.A.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.</td><td>MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.</td><td>Concept(s): No new concept(s) introduced
Students are able to:
• draw the transformed figure using, graph paper, tracing paper, and/or geometry software given a geometric figure and a rotation, reflection, or translation.
• identify the sequence of transformations required to carry one figure onto another.

Learning Goal 5: Draw transformed figures using graph paper, tracing paper, and/or geometry software and identify a sequence of transformations required in order to map one figure onto another.</td><td>Draw the reflection of the following quadrilateral over the line p.
</td></tr>
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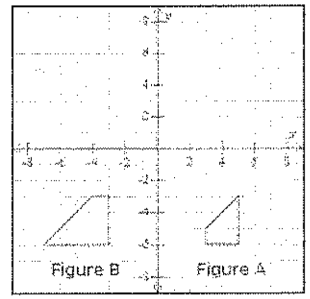
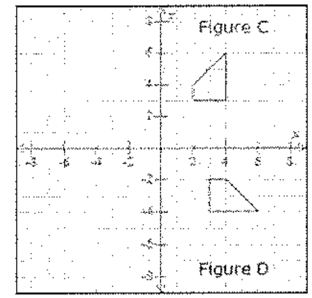
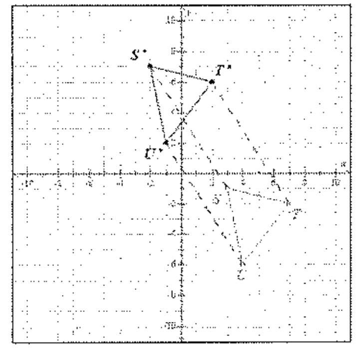
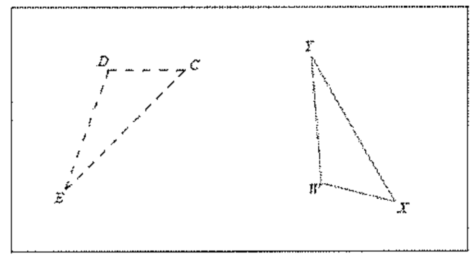
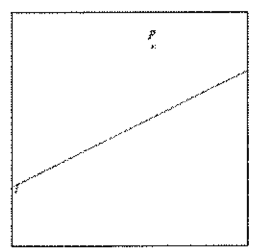
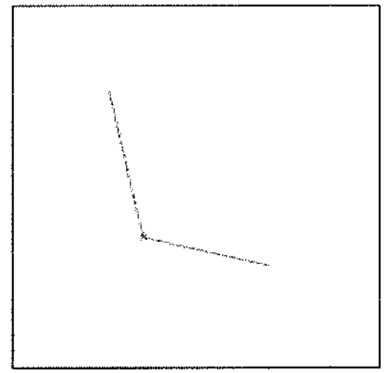
Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.CO.B.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	MP.3 Construct viable arguments and critique the reasoning of others.	Concept(s): • Congruence in terms of rigid motion Students are able to: • predict the outcome of a transformation on a figure. • given a description of the rigid motions, transform figures. • given two figures, decide if they are congruent by applying rigid motions. Learning Goal 6: Use rigid transformations to determine and explain congruence of geometric figures.	 Are Figure A and Figure B congruent? Yes No Which transformation will map Figure A onto Figure B exactly? Translate Figure A to the left 3 units Translate Figure A down 3 units Reflect Figure A over the x-axis Reflect Figure A over the y-axis Rotate Figure A clockwise 180° about the origin Rotate Figure A counterclockwise 90° about the origin None of these  Are Figure C and Figure D congruent? Yes No Which transformation will map Figure C onto Figure D exactly? Translate Figure C to the right 5 units Translate Figure C up 5 units Reflect Figure C over the x-axis Reflect Figure C over the y-axis Rotate Figure C clockwise 90° about the origin Rotate Figure C counterclockwise 180° about the origin None of these

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.CO.B.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): • Triangle congruence in terms of rigid motion Students are able to: • given that two triangles are congruent based on rigid motion, show that corresponding pairs of sides and angles are congruent. • given that corresponding pairs of sides and angles of two triangles are congruent, show, using rigid motion (transformations) that they are congruent. Learning Goal 7: Show and explain that two triangles are congruent by using corresponding pairs of sides and corresponding pairs of angles, and by using rigid motions (transformations).	Translating $\triangle STU$ to the left 5 units and upward 8 units, we get its image $\triangle S'T'U'$.  Note that $\triangle STU$ has vertices $S(3, -1)$, $T(7, -2)$, and $U(4, -6)$. Also, note that $\triangle S'T'U'$ has vertices $S'(-2, 7)$, $T'(2, 6)$, and $U'(-1, 2)$. Complete the following. (a) Find each slope. Give exact answers (not decimal approximations). Slope of $\overline{SS'}$ = ☐ Slope of $\overline{TT'}$ = ☐ Slope of $\overline{UU'}$ = ☐ (b) Find each length. Give exact answers (not decimal approximations). SS' = ☐ units TT' = ☐ units UU' = ☐ units (c) Choose the correct statement about the translation. ○ All points on the original figure moved in the same direction, but not all points moved the same distance. ○ All points on the original figure moved the same distance, but not all points moved in the same direction. ○ All points on the original figure moved the same distance and in the same direction. ○ The points on the original figure didn't all move the same distance and didn't all move in the same direction.

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
■ G.CO.B.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): • Criteria for triangle congruence Students are able to: • show and explain the criteria for Angle-Side-Angle triangle congruence. • show and explain the criteria for Side-Angle-Side triangle congruence. • show and explain the criteria for Side-Side-Side triangle congruence. • explain the relation of the criteria for triangle congruence to congruence in terms of rigid motion. Learning Goal 8: Show and explain how the criteria for triangle congruence extend from the definition of congruence in terms of rigid motion.	For the triangles below, use the tools to move the solid triangle exactly onto the dashed one. Then answer the parts below.  (a) Complete the congruence statements below. $\angle W \cong$ (Choose one) $\angle X \cong$ (Choose one) $\angle Y \cong$ (Choose one) (b) Complete the congruence statements below. $\overline{WX} \cong$ (Choose one) $\overline{XY} \cong$ (Choose one) $\overline{WY} \cong$ (Choose one) (c) Are all three pairs of corresponding angles and all three pairs of corresponding sides congruent? ○ No, and the triangles are not congruent. ○ No, and the triangles are congruent. ○ Yes, and the triangles are not congruent. ○ Yes, and the triangles are congruent.
□ G.CO.D.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software,	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically.	Concept(s): • Congruence underlies formal constructions. Students are able to: • perform formal constructions using a variety of tools and methods including: – copying a segment; – copying an angle;	Use the compass and ruler to construct the line perpendicular to the line l through the point P. 

Unit 1 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* ☐ G.CO.D.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	MP.6 Attend to precision.	– bisecting a segment; – bisecting an angle; – constructing perpendicular lines; – constructing the perpendicular bisector of a line segment; – constructing a line parallel to a given line through a point not on the line; – constructing an equilateral triangle; – constructing a square; – and constructing a regular hexagon inscribed in a circle. • identify the congruencies underlying each construction. Learning Goal 9: Make formal constructions using a variety of tools (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) and methods.	Use the compass and ruler to construct the bisector of the angle given below. 

Vocabulary
Point, Line, Plane, Collinear, Coplanar, Angle, Acute, Obtuse, Right, Straight, Postulate, Segment, Ray, Bisector, Parallel lines, Perpendicular lines, Skew lines, Midpoint Transformation, Translation, Reflection, Rotation, Line of reflection, Symmetry, Center of Rotation, Isometry, Line symmetry, Rotational Symmetry, Center of Symmetry Protractor, Compass

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<tr><th>Content Standards</th><th>Suggested Standards for Mathematical Practice</th><th>Critical Knowledge & Skills</th><th>Examples</th></tr>
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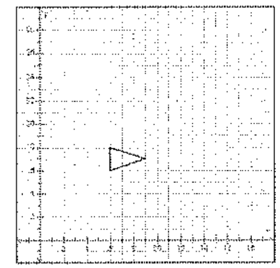
<td>■ G.SRT.A.1. Verify experimentally the properties of dilations given by a center and a scale factor:
G.SRT.A.1a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
G.SRT.A.1b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td>
<td>MP.1 Make sense of problems and persevere in solving them
MP.3 Construct viable arguments and critique the reasoning of others.
MP.5 Use appropriate tools strategically.
MP.8 Look for and express regularity in repeated reasoning.</td>
<td>Concept(s):
• Dilation of a line that passes through the center of dilation results in the same line.
• Dilation of a line that does not pass through the center of dilation results in a line that is parallel to the original line.
• Dilation of a line segment results in a longer line segment when, for scale factor k, |k| is greater than 1.
• Dilation of a line segment results in a shorter line segment when, for scale factor k, |k| is less than 1.
Students are able to:
• perform dilations in order to verify the impact of dilations on lines and line segments.

■ Learning Goal 1: Verify the properties of dilations given by a center and a scale factor.</td>
<td>Draw the image of the following triangle after a dilation centered at the origin with a scale f
</td>
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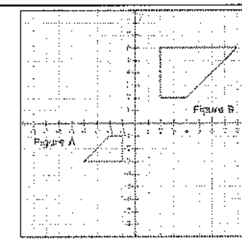
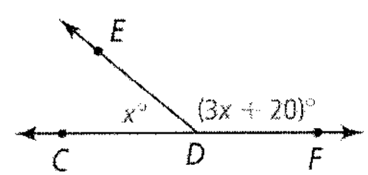
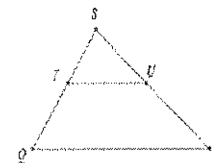
<tr>
<td>■ G.SRT.A.2. Given two figures, use the definition of similarity to decide if they are similar; explain using the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

■ G.SRT.A.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</td>
<td>MP.3 Construct viable arguments and critique the reasoning of others.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.8 Look for and express regularity in repeated reasoning.</td>
<td>Concept(s):
• Similarity transformations are used to determine the similarity of two figures.
Students are able to:
• given two figures, determine if they are similar.
• explain the meaning of similarity for triangles.
• explain AA criterion and its relationship to similarity transformations and properties of triangles

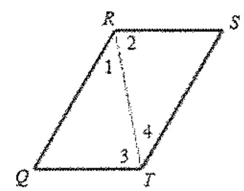
Learning Goal 2: Use the definition of similarity in terms of similarity transformations to decide if two given figures are similar and explain, using similarity transformations, the meaning of triangle similarity.</td>
<td>
(a) Which sequence(s) of transformations will map Figure A onto Figure B exactly? Choose all that apply.
☐ Rotate Figure A clockwise 180° about the origin, and then dilate that result with scale factor 2 centered at the origin.
☐ Dilate Figure A with scale factor 2 centered at the origin, and then rotate that result clockwise 90° about the origin.
☐ Dilate Figure A with scale factor 2 centered at the origin, and then translate that result up 10 units.
☐ Dilate Figure A with scale factor 2 centered at the origin, and then reflect that result over the x-axis.
☐ None of these
(b) Are Figure A and Figure B similar?
○ Yes ○ No</td>
</tr>
</table>

Unit 2 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
■ G.CO.C.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* ■ G.CO.C.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* ■ G.CO.C.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*	MP.3 Construct viable arguments and critique the reasoning of others. MP.6 Attend to precision.	Concept(s): • A formal proof may be represented with a paragraph proof or a two-column proof. Students are able to: • construct and explain proofs of theorems about lines and angles including: – vertical angles are congruent; – angle and segment addition postulate – complementary and supplementary angles – algebraic and midpoint proofs – congruence of alternate interior angles; – congruence of corresponding angles; – and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. • construct and explain proofs of theorems about triangles including: – sum of interior angles of a triangle; – congruence of base angles of an isosceles triangle; – the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; – and the medians of a triangle meet at a point. • construct and explain proofs of theorems about parallelograms including: – opposite sides are congruent; opposite angles are congruent; – the diagonals of a parallelogram bisect each other; – and rectangles are parallelograms with congruent diagonals Learning Goal 4: Construct and explain formal proofs of theorems involving lines, angles, triangles, and parallelograms.	$\frac{1}{2}x - 5 = 10$ Given $2\left(\frac{1}{2}x - 5\right) = 20$ a. ? $x - 10 = 20$ b. ? $x = 30$ c. ? ∠CDE and ∠EDF are supplementary. ∠s that form a linear pair are supplementary $m\angle CDE + m\angle EDF = 180$ a. ? $x + (3x + 20) = 180$ b. ? $4x + 20 = 180$ c. ? $4x = 160$ d. ? $x = 40$ e. ? 

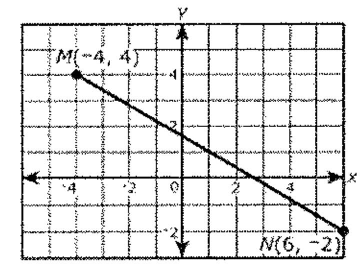
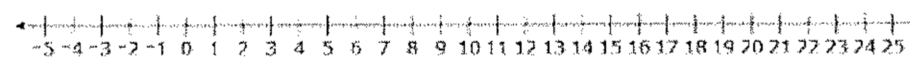
Unit 2 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.SRT.B.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • construct and explain proofs of theorems about triangles including: – a line parallel to one side of a triangle divides the other two sides proportionally; – and the Pythagorean Theorem (using triangle similarity). Learning Goal 5: Prove theorems about triangles.	In ΔQRS, $\overline{QR} \parallel \overline{TU}$. Given that $SQ = 27$, $ST = 12$, and $TU = 20$, find QR. 
G.SRT.B.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	MP.7 Look for and make use of structure.	Concept(s): • Corresponding parts of congruent triangles are congruent (CPCTC). Students are able to: • prove geometric relationships in figures using criteria for triangle congruence. • solve problems using triangle congruence criteria (SSS, ASA, SAS, HL). • solve problems using triangle similarity criteria (AA). Learning Goal 6: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Use the given information to prove that $\overline{VU} \cong \overline{SR}$.  Given: $\overline{UR}$ bisects $\overline{VS}$ $\overline{VS} \perp \overline{VU}$ $\overline{VS} \perp \overline{SR}$ Prove: $\overline{VU} \cong \overline{SR}$

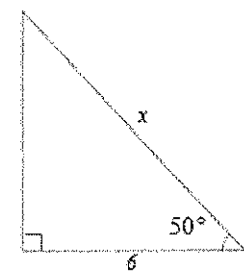
Vocabulary
Property of equality, addition property, subtraction property, multiplication property, division property, substitution property, symmetric property, reflexive property, transitive property, distributive property, property of congruence, two column proof, corresponding angles and their converse, alternate interior angles and their converse, alternate exterior angles and their converse, same side interior angles and their converse, triangle angle sum theorem, exterior angle, remote interior angles, SSS, SAS, ASA, AAS, HL, isosceles triangle, equilateral triangle, equiangular triangle, CPCTC, triangle mid-segment, perpendicular bisector, median, centroid, altitude, parallelogram.

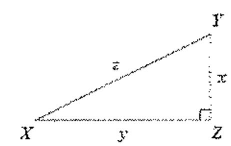
Unit 3 Geometry

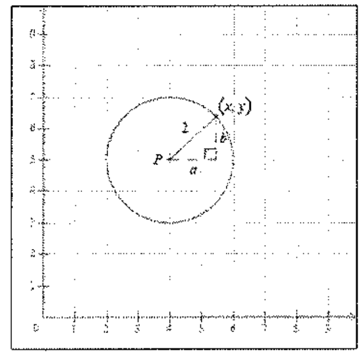
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	Examples
G.GPE.B.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle;*	MP.3 Construct viable arguments and critique the reasoning of others.	Concept(s): No new concept(s) introduced Students are able to: • Use coordinates to prove geometric theorems including: – prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle (or other quadrilateral); Learning Goal 1: Use coordinates to prove simple geometric theorems algebraically.	Use the given information to complete the proof of the following theorem. *If opposite sides of a quadrilateral are congruent, then it is a parallelogram.* By definition, a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Use this definition in your proof.  Given: $\overline{QR} \cong \overline{ST}$ $\overline{QT} \cong \overline{RS}$ Prove: $QRST$ is a parallelogram

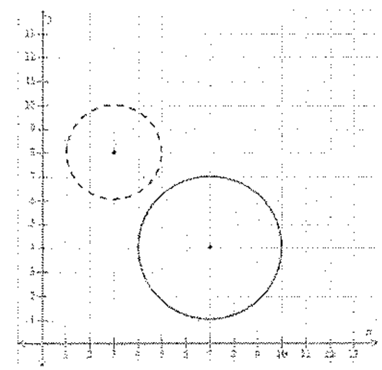
	Statement	Reason	Line(s) Used
1	$\overline{QR} \cong \overline{ST}$	Given	
2	$\overline{QT} \cong \overline{RS}$	Given	
3	$\overline{\square} \cong \overline{\square}$	Reflexive Property	
4	$\triangle\square \cong \triangle\square$	***Reason ?***	
5	$\angle 1 \cong \angle\square$	CPCTC Property	4
6	$\angle 2 \cong \angle\square$	CPCTC Property	4
7	$\overline{QR} \parallel \overline{\square}$	***Reason ?***	
8	$\overline{QT} \parallel \overline{\square}$	***Reason ?***	
9	$QRST$ is a parallelogram	***Reason ?***	

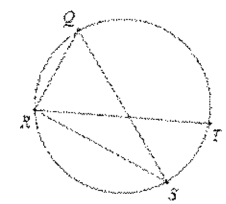
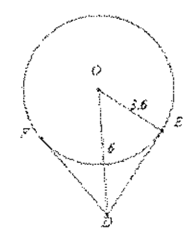
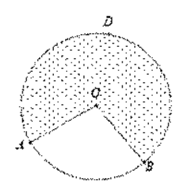
Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.GPE.B.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. G.GPE.B.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • locate the point on a directed line segment that creates two segments of a given ratio. • find perimeters of polygons using coordinates, the Pythagorean theorem and the distance formula. • find areas of triangle and rectangles using coordinates. Learning Goal 3: Find the point on a directed line segment between two given points that partitions the segment in a given ratio and use coordinates to compute perimeters of polygons and areas of triangles and rectangles.	The diagram shows $\overline{MN}$ graphed on a coordinate plane  Point P lies on $\overline{MN}$ and is $\frac{3}{4}$ of the way from M to N. What are the coordinates of point P? Enter your answer in the space provided. Enter **only** your answer. (,) Point A is located at −3, and point B is located at 19. Select a point on the number line between A and B such that the distance from A to the point is $\frac{3}{11}$ of the distance from A to B. Select a place on the number line to plot the point 

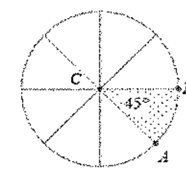
Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.SRT.C.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	MP.7 Look for and make use of structure.	Concept(s): • Side ratios in right triangles are properties of the angles in the triangle. Students are able to: • show and explain that definitions for trigonometric ratios derive from similarity of right triangles. Learning Goal 4: Show and explain that definitions for trigonometric ratios derive from similarity of right triangles.	Solve for x in the triangle. Round your answer to the nearest tenth. 

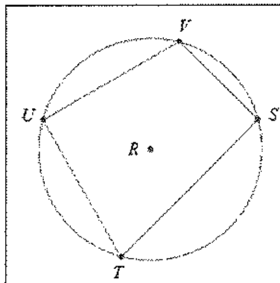
Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
■ G.SRT.C.7. Explain and use the relationship between the sine and cosine of complementary angles ■ G.SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): • Relationship between sine and cosine of complementary angles Students are able to: • determine and compare sine and cosine ratios of complementary angles in a right triangle. • solve right triangles (determine all angle measures and all side lengths) using trigonometric ratios and the Pythagorean Theorem. Learning Goal 5: Explain and use the relationship between the sine and cosine of complementary angles; use trigonometric ratios and the Pythagorean Theorem to compute all angle measures and side lengths of triangles in applied problems.	The figure below is a right triangle with side lengths x, y, and z. Suppose that $m\angle X$ does not equal $m\angle Y$.  Complete the following. **Part 1:** Use x, y, and z to fill in the blanks. Make sure to use the appropriate upper-case or lower-case letters. $\sin X = \frac{\square}{\square}$ $\sin Y = \frac{\square}{\square}$ $\cos X = \frac{\square}{\square}$ $\cos Y = \frac{\square}{\square}$ **Part 2:** In $\triangle XYZ$, $\angle X$ and $\angle Y$ are (Choose one) ▾ **Part 3:** Select *all* of the true statements. ☐ $\sin X = \sin Y$ ☐ $\sin X = \cos Y$ ☐ $\cos X = \cos Y$ ☐ $\cos X = \sin Y$ ☐ None of the above is true. **Part 4:** Fill in the blank. $\sin(58°) = \cos(\square°)$

Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
◎ G.GPE.A.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to: • given the center and radius, derive the equation of a circle (using the Pythagorean Theorem). • given an equation of a circle in any form, use the method of completing the square to determine the center and radius of the circle. Learning Goal 6: Derive the equation of a circle of given the center and radius using the Pythagorean Theorem. Given an equation, complete the square to find the center and radius of the circle.	The circle below has center P. The point (x, y) is on the circle as shown.  (a) Find the following. Radius: ☐ units Center: (☐, ☐) Value of a: (Choose one) ▾ Value of b: (Choose one) ▾ (b) Use the Pythagorean Theorem to write an equation relating the side lengths of the right triangle. Write your answer in terms of x and y (with no other letters). $(\square)^2 + (\square)^2 = (\square)^2$

Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
◎ G.C.A.1. Prove that all circles are similar.	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically.	Concept(s): • Similarity of all circles Students are able to: • construct a formal proof of the similarity of all circles. Learning Goal 7: Prove that all circles are similar	In the figure below, the solid circle has center $(7, 4)$ and radius 3. The dashed circle has center $(3, 8)$ and radius 2. Use the transformation tools given to move the solid circle exactly onto the dashed circle. Then answer the parts below.  (a) Fill in the blanks to describe the transformations necessary to move the solid circle exactly onto the dashed circle. Translate the solid circle [Choose one ▾] by [ ] unit(s) and [Choose one ▾] by [ ] unit(s). Dilate the solid circle by a scale factor of [ ]. (b) Are the original solid circle and the dashed circle similar? ○ Yes ○ No

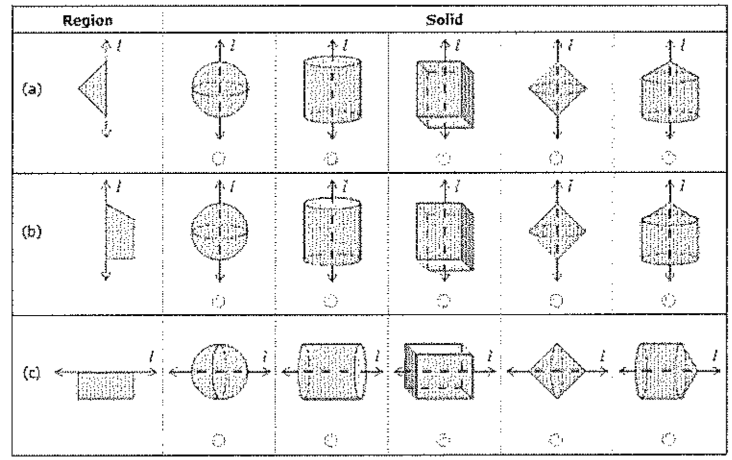
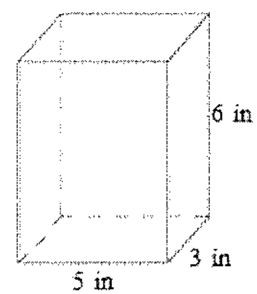
Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
◎ G.C.A.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools strategically.	Concept(s): No new concept(s) introduced Students are able to: • use the relationship between inscribed angles, radii and chords to solve problems. • use the relationship between central, inscribed, and circumscribed angles to solve problems. • identify inscribed angles on a diameter as right angles. • identify the radius of a circle as perpendicular to the tangent where the radius intersects the circle. • find arc length and area of a sector Learning Goal 8: Identify and describe relationships among inscribed angles, radii, and chords; use these relationships to solve problems.	In the circle below, $\overline{QS}$ is a diameter. Suppose $m\widehat{QR} = 62°$ and $m\angle QRT = 66°$. Find the following. (a) $m\angle RQS$ (b) $m\angle SRT$  In the figure below, the segments $\overline{DE}$ and $\overline{DF}$ are tangent to the circle centered at O. Given that $OE = 3.6$ and $OD = 6$, find DF.  The circle below has center O, and its radius is 7 ft. Given that $m\angle AOB = 100°$, find the length of the arc $\widehat{ADB}$ and the area of the shaded region. Give exact answers in terms of π, and be sure to include the correct units in your answer. 

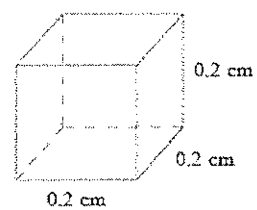
Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
◎ G.C.B.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique he reasoning of others.	Concept(s): • A proportional relationship exists between the length of an arc that is intercepted by an angle and the radius of the circle. Students are able to: • use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius. • define radian measure of an angle as the constant of proportionality when the length of the arc intercepted by an angle is proportional to the radius. • derive the formula for the area of a sector. • compute arc lengths and areas of sectors of circles. Learning Goal 7: Find arc lengths and areas of sectors of circles; use similarity to show that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector.	The circle below with center C is divided into 8 equal slices. Central angle $\angle ACB$ intercepts $\overset{\frown}{AB}$, forming the shaded sector.  Complete the statements below. Give your answers as exact values, not decimal approximations. Complete the statements below. Give your answers as exact values, not decimal approximations. (a) From the figure we see that $m\angle ACB = 45°$. The measure of $\angle ACB$ is (Choose one) the measure of $\overset{\frown}{AB}$. So, $m\overset{\frown}{AB} = \square°$. This gives the following equation. $\frac{m\overset{\frown}{AB}}{360°} = \square$ (b) Find the ratio of the area of the sector to the area of the circle. $\frac{\text{Area of sector}}{\text{Area of circle}} = \square$ (c) Choose the equation that is true. ○ $\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{360°}{m\overset{\frown}{AB}}$ ○ $\frac{\text{Area of sector}}{\text{Area of circle}} = 8 \cdot \frac{m\overset{\frown}{AB}}{360°}$ ○ $\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{m\overset{\frown}{AB}}{360°}$ ○ $\frac{\text{Area of sector}}{\text{Area of circle}} = m\overset{\frown}{AB}$ (d) Using πr^2 for the area of the circle, choose the equation that gives a formula for the area of the sector. ○ Area of sector $= \frac{m\overset{\frown}{AB}}{360°} \cdot 8\pi r^2$ ○ Area of sector $= \frac{m\overset{\frown}{AB}}{360°} \cdot \pi r^2$ ○ Area of sector $= m\overset{\frown}{AB} \cdot \pi r^2$ ○ Area of sector $= \frac{360°}{m\overset{\frown}{AB}} \cdot \pi r^2$

Unit 3 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
◎ G.C.A.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically	Concept(s): No new concept(s) introduced Students are able to: • construct the inscribed circle of a triangle. • construct the circumscribed circle of a triangle. • prove properties of the angles of a quadrilateral that is inscribed in a circle. Learning Goal 9: Prove the properties of angles for a quadrilateral inscribed in a circle and construct inscribed and circumscribed circles of a triangle using geometric tools and geometric software.	Quadrilateral $STUV$ is inscribed in circle R. Complete the following.  (a) Choose the correct expression to finish each statement. $m\angle T$ = (Choose one) ▼ $m\angle V$ = (Choose one) ▼ (b) Choose the correct expressions for the sum of $m\angle T$ and $m\angle V$. $m\angle T + m\angle V$ = (Choose one) ▼ = (Choose one) ▼ (c) Fill in the blank with the correct number. $m\angle T + m\angle V$ = ☐° (d) From the steps above, classify $\angle T$ and $\angle V$. ○ $\angle T$ and $\angle V$ are supplementary. ○ $\angle T$ and $\angle V$ are neither supplementary nor complementary. ○ $\angle T$ and $\angle V$ are complementary.

Vocabulary

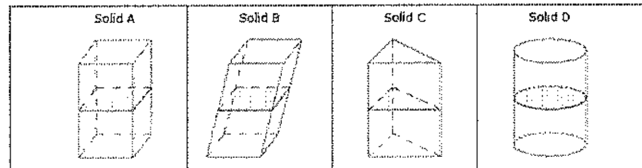
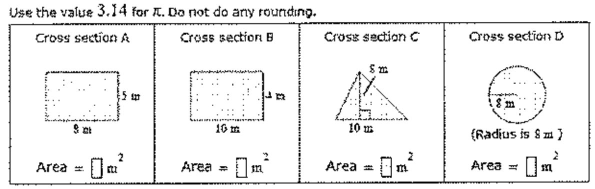
Ratio, proportion, cross products property, similar figures, similar polygons, scale factor, scale drawing, geometric mean, Pythagorean triple, trigonometric ratios, sine, cosine, tangent, angle of elevation, angle of depression, circle, center, diameter, radius, congruent circles, central angle, semicircle, minor arc, major arc, adjacent arcs, circumference, pi, arc length congruent arcs.

Unit 4 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
■ G.MG.A.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder. ◎ G.GMD.A.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ◎ G.GMD.B.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): • Real-world objects can be described, approximately, using geometric shapes, their measures, and their properties. Students are able to: • identify cross-sections of three dimensional objects. • identify three-dimensional objects generated by rotation of two-dimensional objects. • solve problems using volume formulas for cylinders, pyramids, cones, and spheres. • model real-world objects with geometric shapes. • describe the measures and properties of geometric shapes that best represent a real-world object. Learning Goal 1: Model real-world objects with geometric shapes based upon their measures and properties, and solve problems using volume formulas for cylinders, pyramids, cones, and spheres. Identify cross-sections, three-dimensional figures, and identify three-dimensional objects created by the rotation of two-dimensional objects.	Each region below is being rotated about the line l. For each region, identify the solid generated by the rotation.  Find the volume of the rectangular prism. 

Unit 4 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
■ G.MG.A.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • model real-world situations, applying density concepts based on area. • model real-world situations, applying density concepts based on volume. Learning Goal 2: Apply concepts of density based on area and volume in modeling situations.	Answer the questions below. Give exact integer or decimal answers. Be sure to include the correct units in your answers. (a) An object has a mass of 768 kg and a volume of 240 m^3. What is the density of the object? Density = ▯ (b) Another object has a volume of 25 m^3 and a density of 20 $\frac{kg}{m^3}$. What is the mass of the object? Mass = ▯ A solid object in the shape of a rectangular prism is shown below. If its density is 5 $\frac{g}{cm^3}$, what is its mass? Give an exact integer or decimal answer. Be sure to include the correct unit in your answer. 

Unit 4 Geometry			
Content Standards	**Suggested Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
G.MG.A.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • design objects or structures satisfying physical constraints • design objects or structures to minimize cost. • solve design problems. Learning Goal 3: Solve design problems using geometric methods	Lisa has an online jewelry shop where she sells earrings and necklaces. She sells earrings for $30 and necklaces for $40. It takes 30 minutes to make a pair of earrings and 1 hour to make a necklace, and, since Lisa is a math tutor, she only has 10 hours a week to make jewelry. In addition, she only has enough materials to make 15 total jewelry items per week. She makes a profit of $15 on each pair of earrings and $20 on each necklace. How many pairs of earrings and necklaces should Lisa make each week in order to maximize her profit, assuming she sells all her jewelry? Define the variables, write an inequality for this situation, and graph the solutions to the inequality.

Unit 4 Geometry

Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	Examples
◎ G.GMD.A.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	MP.3 Construct viable arguments and critique the reasoning of others. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to: • construct viable dissection arguments and informal limit arguments. • apply Cavalieri's principle. • construct an informal argument for the formula for the circumference of a circle. • construct an informal argument for the formula for the area of a circle. • construct an informal argument for the formula for the volume of a cylinder, pyramid, and cone. Learning Goal 4: Using dissection arguments, Cavalieri's principle, and informal limit arguments, develop informal arguments for formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	A right rectangular prism, a slanted rectangular prism, a right triangular prism, and a right cylinder are shown below. A plane parallel to the bases crosses the solids at the same level. The resulting cross sections (shaded) are shown. Answer the following questions. Note that the figures are not drawn to scale.  (a) Find the areas of the cross sections. Use the value 3.14 for π. Do not do any rounding.  (b) If the heights of the solids are the same, which of these solids have the same volume as Solid A? Check all that apply. ☐ Solid B ☐ Solid C ☐ Solid D ☐ None of these (c) Which of the following was most closely used in getting the answer to part (b)? ○ As long as solids have the same cross sectional area at a certain level, they have the same volume. ○ Even if solids have the same height and the same cross sectional area at every level, they don't necessarily have the same volume. ○ If solids have the same height and the same cross sectional area at every level, then they have the same volume. ○ As long as solids have the same height, they have the same volume.

Vocabulary

Polyhedron, face, edge, vertex, Euler's formula, cross section, surface area, prism, base, lateral face, altitude, height, right prism, oblique prism, lateral area, cylinder, pyramid, slant height, cone, volume, Cavalieri's principle, sphere, hemisphere, similar solids, density, constraint, linear optimization, cost.