TOWNSHIP OF UNION PUBLIC SCHOOLS

UHS Algebra I
Curriculum Guide 2017

Mission Statement

The mission of the Township of Union Public Schools is to build on the foundations of honesty, excellence, integrity, strong family, and community partnerships. We promote a supportive learning environment where every student is challenged, inspired, empowered, and respected as diverse learners. Through cultivation of students' intellectual curiosity, skills and knowledge, our students can achieve academically and socially, and contribute as responsible and productive citizens of our global community.

Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is to formulate a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

Algebra I provides a formal development of the algebraic skills and concepts necessary for students to succeed in advanced courses. In particular, the instructional program in this course provides for the use of algebraic skills in a wide range of problem-solving situations. The concept of function is emphasized throughout the course. Topics include: (1) operations with real numbers, (2) linear equations and inequalities, (3) relations and functions, (4) polynomials, (5) algebraic fractions, and (6) nonlinear equations.

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 1	■ N.Q.A.1 ■ N.Q.A.2 ■ N.Q.A.3 ■ A.REI.B.3 ■ A.REI.A.1 ■ A.CED.A.4 ■ A.SSE.A.1 ■ A.CED.A.1 ■ F.IF.A.2 ■ A.REI.A.1 ■ A.CED.A.2 ■ A.REI.D.10 □ S.ID.B.6 ■ S.ID.C.7 ■ S.ID.C.8 ■ S.ID.C.9 ■ A.REI.D.11	• Reason quantitatively and use units to solve problems • Solve [linear] equations and inequalities in one variable • Understand solving equations as a process of reasoning and explain the reasoning • Create equations that describe numbers or relationships • Interpret the structure of expressions • Represent and solve equations graphically • Summarize, represent, and interpret data on quantitative variables. Interpret linear models • Understand the concept of a function and use function notation	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 1: Suggested Educational Resources	N.Q.A.1 Runners' World **N.Q.A.2 Giving Raises** N.Q.A.3 Calories in a Sports Drink A.REI.B.3, A.REI.A.1 Reasoning with linear inequalities A.CED.A.4 Equations and Formulas	A.SSE.A.1 Kitchen Floor Tiles A.CED.A.1 Planes and wheat A-CED.A.1 Paying the rent **A.REI.A.1 Zero Product Property 1** A.CED.A.2 Clea on an Escalator S.ID.B.6,S.ID.C.7-9 Coffee and Crime	
Unit 2	◎ A.REI.C.6 ■ A.CED.A.3 ◎ A.REI.C.5 ■ A.REI.D.12 ■ F.IF.A.1 □ F.LE.A.1 □ F.LE.A.2 ■ F.IF.A.3 □ F.BF.A.1 ■ A.SSE.A.1 □ A.SSE.B.3 ■ F.IF.B.4 □ F.LE.B.5 ■ F.IF.B.5 ■ F.IF.B.6 □ F.IF.C.9 □ F.IF.C.7	• Solve linear systems of equations • Create equations that describe numbers or relationships • Interpret the structure of expressions • Represent and solve equations and inequalities graphically • Construct & compare linear & exponential models • Interpret expressions for functions in terms of the situation • Build a function that models a relationship between two quantities • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations	
Unit 2: Suggested Educational Resources	A.REI.C.6 Cash Box A.CED.A.3 Dimes and Quarters A.REI.C.5 Solving Two Equations in Two Unknowns A.REI.D.12 Fishing Adventures 3 F.IF.A.1 The Parking Lot F.IF.A.2 Yam in the Oven F.LE.A.1 Finding Linear and Exponential Models F.LE.A.2 Interesting Interest Rates	F.BF.A.1a Skeleton Tower A.SSE.A.1 Mixing Candies F.IF.B.4 Warming and Cooling F.IF.B.4, F.IF.B.5 Average Cost F.LE.B.5 US Population 1982-1988 F.IF.B.6 Temperature Change F.IF.C.7b Bank Account Balance	

<table>
<tr><th>Overview</th><th>Standards for Mathematical Content</th><th>Unit Focus</th><th>Standards for Mathematical Practice</th></tr>
<tr><td><u>Unit 3</u></td><td>■ A.APR.A.1
■ A.SSE.A.2
■ A.REI.B.4
■ A.CED.A.1
■ F.IF.B.4*
■ F.IF.B.5*
□ A.SSE.B.3
□ F.BF.A.1
□ F.IF.C.7*
□ F.IF.C.8*
□ F.IF.C.9*
■ F.IF.B.6
□ F.LE.A.3
◎ F.BF.B.3
■ A.REI.D.11
□ A.APR.B.3
◎ N.RN.B.3</td><td>• Perform arithmetic operations on polynomials
• Understand the relationship between zeros and factors
• Interpret the structure of expressions
• Solve equations and inequalities in one variable
• Create equations that describe numbers or relationships
• Interpret functions that arise in applications in terms of the context
• Represent and solve equations and inequalities graphically
• Build a function that models a relationship between two quantities
• Construct & compare linear, quadratic, & exponential models
• Build new functions from existing functions
• Analyze functions using different representations
Use properties of rational and irrational numbers</td><td rowspan="4">MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments & critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.</td></tr>
<tr><td>Unit 3: Suggested Educational Resources</td><td colspan="2"><u>A.APR.A.1 Powers of 11</u>
<u>A.SSE.A.2 Equivalent Expressions</u>
<u>A.REI.B.4 Visualizing Completing the Square</u>
<u>A.REI.B.4 Braking Distance</u>
<u>A.REI.B.4 Two Squares are Equal</u>
<u>F.IF.B.4 Words – Tables - Graphs</u>
<u>F.IF.B.5 The restaurant</u>
<u>A.SSE.B.3 Profit of a company</u>
<u>A.SSE.B.3 Rewriting a Quadratic Expression</u>
<u>F.IF.C.7a Graphs of Quadratic Functions</u>
<u>F.IF.C.8a Springboard Dive</u>
<u>F.IF.C.8a Which Function?</u>
<u>F.IF.B.9 Throwing Baseballs</u>
<u>F.IF.B.6 Mathemafish Population</u>
<u>F.LE.A.3 Population and Food Supply</u>
<u>F.BF.B.3 Identifying Even and Odd Functions</u>
<u>F.BF.B.3 Transforming the graph of a function</u>
<u>A.REI.D.11 Introduction to Polynomials – College Fund</u>
<u>A.APR.B.3 Graphing from Factors 1</u>
<u>N.RN.B.3 Operations with Rational and Irrational Numbers</u></td></tr>
<tr><td><u>Unit 4</u></td><td>◎ S.ID.A.1
◎ S.ID.A.2
◎ S.ID.A.3
□ S.ID.B.5
□ S.ID.B.6
■ F.IF.B.4*
■ F.IF.B.5*
■ 8.G.B.8*</td><td>• Summarize, represent, and interpret data on a single count or measurement variable
• Summarize, represent, and interpret data on two categorical and quantitative variables
Interpret functions that arise in applications in terms of the context
• Understand and apply the Pythagorean Theorem</td></tr>
<tr><td>Unit 4: Suggested Educational Resources</td><td colspan="2"><u>S.ID.A.1-3 Haircut Costs</u>
<u>S.ID.A.1-3 Speed Trap</u>
<u>S.ID.A.2-3 Measuring Variability in a Data Set</u>
<u>S.ID.A.3 Identifying Outliers</u>
<u>S.ID.B.5 Support for a Longer School Day?</u>
<u>S.ID.B.6 Laptop Battery Charge 2</u>
<u>F.IF.B.4 The Aquarium</u>
<u>F.IF.B.4 Containers</u>
<u>F.IF.B.4-5 The Canoe Trip, Variation 2</u>
<u>8.G.B.8 Finding the distance between points</u></td></tr>
</table>

Unit 1 Algebra I			
Content & Practice Standards	**Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
☐ N.Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays. ☐ N.Q.A.2. Define appropriate quantities for the purpose of descriptive modeling. ☐ N.Q.A.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	MP.1 Make sense of problems and persevere in solving them. MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.	Concept(s): • Units are associated with variables in expressions and equations in context. • Quantities may be used to model attributes of real world situations. • Measurement tools have an inherent amount of uncertainty in measurement. Students are able to: • use units to understand real world problems. • use units to guide the solution of multi-step real world problems (e.g. dimensional analysis). • choose and interpret units while using formulas to solve problems. • identify and define appropriate quantities for descriptive modeling. • choose a level of accuracy when reporting measurement quantities. Learning Goal 1: Solve multi-step problems, using units to guide the solution, interpreting units consistently in formulas and choosing an appropriate level of accuracy on measurement quantities. Develop descriptive models by defining appropriate quantities.	In my math class, 28 students are each given 3 pencils. If there are 8 pencils in one package, priced at $1.88 per package, what is the total cost of giving away pencils? Sally James was pulled over on her way from Springfield to Union by an officer claiming she was speeding. The speed limit is 65 mi/hr and Sally had traveled 97 km in 102 minutes. How fast was Sally's average speed? Does she deserve a ticket?
■ A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context. A.SSE.A.1a. Interpret parts of an expression, such as	MP.1 Make sense of problems and persevere in solving them. MP 2 Reason abstractly and quantitatively.	Concept(s): No new concept(s) introduced Students are able to: • identify different parts of an expression, including terms, factors and constants.	Give an example of two like terms and two unlike terms. Explain why they would or would not be classified as like terms.

terms, factors, and coefficients.		• explain the meaning of parts of an expression in context. Learning Goal 3: Interpret terms, factors, coefficients, and other parts of expressions in terms of a context.	
■ A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. ■ A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): • Equations and inequalities describe relationships. • Equations can represent real-world and mathematical problems. Students are able to: • identify and describe relationships between quantities in word problems. • create linear equations in one variable. • create linear inequalities in one variable. • use equations and inequalities to solve real world problems. • explain each step in the solution process. Learning Goal 4: Create linear equations and inequalities in one variable and use them in contextual situations to solve problems. Justify each step in the process and the solution.	Jennifer had $30 to spend on herself. She spent 1/5 of the money on a sandwich, 1/6 for a ticket to a museum, and 1/2 of it on a book. How much money does Jennifer have left over? Tim is choosing between two cell phone plans that offer the same amount of free minutes. Sprint's plan charges $39.99 per month with additional minutes costing $0.45. Verizon's plan costs $44.99 with additional minutes at $0.40. How many additional minutes, a, will it take for the two plans to cost the same?
■ A.REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. ■ A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the	MP 2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s). • Literal equations can be rearranged using the properties of equality. Students are able to. • solve linear equations with coefficients represented by letters in one variable. • use the properties of equality to justify steps in solving linear equations. • solve linear inequalities in one variable. • rearrange linear formulas and literal equations,	Rewrite the following formula to highlight the variable "h" $A = \frac{(b_1 + b_2) \cdot h}{2}$ Solve the following inequality for *y*, where *a*, *b*, and *c* are positive real numbers. Show all work and justify each step in the work with

original equation has a solution. Construct a viable argument to justify a solution method. A.CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.		isolating a specific variable. Learning Goal 2. Solve linear equations and inequalities in one variable (including literal equations); justify each step in the process.	mathematical reasoning. $ax - by > c$ Solve the following equation. Show all work and justify each step in the work with a mathematical reason. $\frac{1}{3}(2x-5)-2=\frac{1}{2}(x-2)$ Solve the following inequality $3(x-4) \le 8x+13$ for x
■ A.CED.A.2. Create equations in two or more variables to represent relationships between quantities; Graph equations on coordinate axes with labels and scales. ■ F.IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. □ N.Q.A.1. Use units as a way to understand problems and to guide the solution of multi-step problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays. A.REI.D.10. Understand that the graph of an	MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): • Equations represent quantitative relationships. Students are able to: • create linear equations in two variables, including those from a context. • select appropriate scales for constructing a graph. • interpret the origin in graphs. • graph equations on coordinate axes, including labels and scales. • identify and describe the solutions in the graph of an equation. Learning Goal 5: Create linear equations in two variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	You are visiting Baltimore, MD and a taxi company charges a flat fee of \$3.00 for using the taxi and \$0.75 per mile. A. Write an equation that you could use to find the cost of the taxi ride in Baltimore, MD. Let x represent the number of miles and y represent the total cost. B. How much would a taxi ride for 8 miles cost? C. If a taxi ride cost \$15, how many miles did the taxi travel? While on vacation in Washington DC, the cab ride for the Dulles airport to the hotel is 15 miles. The total cost of the cab ride was \$25.50. The cabbie charges \$1.50 per mile for the entire trip. A. Write an equation to that can be used to determine how much a cab ride would cost anywhere in Washington DC. B. What is the flat rate of the cab

<table>
<tr>
<td>equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Focus on linear equations.]</td>
<td></td>
<td></td>
<td>ride?
C. How much does it cost to travel 7 miles in a cab?

Which of the following statements is NOT true of the origin?
a. The origin is at the point (0, 0).
b. The origin is where the x-axis and y-axis intersect.
c. The origin is where x = 0 and y = 0.

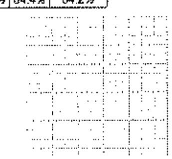
The origin is where x is greater than y</td>
</tr>
<tr>
<td>☐ S.ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
S.ID.B.6a. Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.B.6c. Fit a linear function for a scatter plot that suggests a linear association.</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.</td>
<td>Concept(s):
• Scatter plots represent the relationship between two variables.
• Scatter plots can be used to determine the nature of the association between the variables.
• Linear models may be developed by fitting a linear function to approximately linear data.
• The correlation coefficient represents the strength of a linear association.
Students are able to:
• distinguish linear models representing approximately linear data from linear equations representing "perfectly" linear relationships.
• create a scatter plot and sketch a line of best fit.
• fit a linear function to data using technology.
• solve problems using prediction equations.
• interpret the slope and the intercepts of the linear model in context.
• determine the correlation coefficient for the linear model using technology.
• determine the direction and strength of the linear</td>
<td>6. Make a scatter plot of the data, and draw a line of best fit. Then use the data to predict the percentage of American homeowners in 1985.

Percent of Americans Owning Homes
<table>
<tr><td>Year</td><td>1950</td><td>1960</td><td>1970</td><td>1980</td><td>1990</td></tr>
<tr><td>Percent</td><td>55.0%</td><td>61.9%</td><td>62.9%</td><td>64.4%</td><td>64.2%</td></tr>
</table>
Prediction: ___________


The graph shows the altitude of an airplane as it comes in for a landing. In comparing the time to the altitude, find the rate of change and distinguish the correlation.
<table>
<tr><td>Time(seconds)</td><td>Altitude (feet)</td></tr>
<tr><td>0</td><td>30,000</td></tr>
<tr><td>5</td><td>28,192</td></tr>
<tr><td>10</td><td>25,962</td></tr>
<tr><td>15</td><td>24,450</td></tr>
<tr><td>20</td><td>21,320</td></tr>
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</td>
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</table>

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<tr>
<td>S.ID.C.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.C.9. Distinguish between correlation and causation.</td>
<td></td>
<td>association between two variables.

Learning Goal 6: Represent data on a scatter plot, describe how the variables are related and use technology to fit a function to data.
Learning Goal 7: Interpret the slope, intercept, and correlation coefficient of a data set of a linear model; distinguish between correlation and causation.</td>
<td><table><tr><td>25</td><td>18,780</td></tr><tr><td>30</td><td>15,256</td></tr></table></td>
</tr>
<tr>
<td>A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [Focus on linear equations.]</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.5 Use appropriate tools strategically.</td>
<td>Concept(s):
• y = f(x), y=g(x) represent a system of equations.
• Systems of equations can be solved graphically (8.EE.C.8).
Students are able to:
• explain the relationship between the x-coordinate of a point of intersection and the solution to the equation f(x) = g(x) for linear equations y = f(x) and y = g(x).
• find approximate solutions to the system by making a table of values, graphing, and finding successive approximations.

Learning Goal 8: Explain why the solutions of the equation f(x) = g(x) are the x-coordinates of the points where the graphs of the linear equations y=f(x) and y=g(x) intersect. ** function notation is not introduced here

Learning Goal 9: Find approximate solutions of f(x) = g(x), where f(x) and g(x) are linear functions, by making a table of values, using technology to graph and finding successive approximations.</td>
<td>For the functions defined below, fill in the table of values and circle the row of the table that indicates the solution to f(x) = g(x). Then give the solution to f(x) = g(x).

f(x) = 3x – 8, g(x) = 0.5x + 7
<table><tr><th>x</th><th>f(x)</th><th>g(x)</th></tr><tr><td>0</td><td></td><td></td></tr><tr><td>1</td><td></td><td></td></tr><tr><td>2</td><td></td><td></td></tr><tr><td>3</td><td></td><td></td></tr><tr><td>4</td><td></td><td></td></tr><tr><td>5</td><td></td><td></td></tr><tr><td>6</td><td></td><td></td></tr><tr><td>7</td><td></td><td></td></tr><tr><td>8</td><td></td><td></td></tr></table></td>
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				9			
				10			

Unit 1 Vocabulary

Variable, algebraic expression, equation, evaluate, simplify, exponent, base, power, rational number, irrational number, real numbers, inequality, opposites, absolute value, coordinate plane, coordinates, ordered pair, function, function rule, domain, range, dependent variable, independent variable, scatter plot, correlation, line of best fit, measures of central tendency, additive inverse, matrix, multiplicative inverse, reciprocal, term, coefficient, equivalent equations, solution, consecutive integers, equivalent inequalities, relation, vertical-line test, function notation, continuous data, discrete data, direct variation, inverse variation, inductive reasoning, conjecture, rate of change, slope, linear function, linear equation, slope-intercept form, standard form, y-intercept, x-intercept, point-slope form, parallel and perpendicular lines.

Unit 2 Algebra I	

Content & Practice Standards	Standards for Mathematical Practice	Critical Knowledge & Skills	Examples
◎ A.REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. ■ A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ◎ A.REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	MP.1 Make sense of problems and persevere in solving them. MP 2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics.	Concept(s): • Systems of equations can be solved exactly (algebraically) and approximately (graphically). Students are able to: • identify and define variables representing essential features for the model. • model real world situations by creating a system of linear equations. • solve systems of linear equations using the elimination or substitution method. • solve systems of linear equations by graphing. • interpret the solution(s) in context. Learning Goal 1: Solve multistep contextual problems by identifying variables, writing equations, and solving systems of linear equations in two variables algebraically and graphically.	A garden supply store sells two types of lawn mowers. Total sales of mowers for the year were $8,379.70. the total number of mowers sold was 30. The small mowers cost $249.99 and the large mowers cost $329.99. a. Write two equations clearly defining the variables to represent the above. b. Find the number of each type of lawn mower sold. A.REI.C.6 Cash Box Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it. She said, "I wonder whether the dollar belongs inside the cash box or not." The price of tickets for the dance was 1 ticket for $5 (for individuals) or 2 tickets for $8 (for couples). She looked inside the cash box and found $200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?
■ A.REI.D.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. ■ A.CED.A.3. Represent constraints by equations or inequalities, and by systems	MP.1 Make sense of problems and persevere in solving them. MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools	Concept(s): No new concept(s) introduced Students are able to: • model real world situations by creating a system of linear inequalities given a context. • interpret the solution(s) in context. Learning Goal 2: Graph linear inequalities and systems of linear inequalities in two variables and explain that the solution to the system of inequalities is the	**A clothing manufacturer has 1,000 yd. of cotton to make shirts and pajamas. A shirt requires 1 yd. of fabric, and a pair of pajamas requires 2 yd. of fabric. It takes 2 hr. to make a shirt and 3 hr. to make the pajamas, and there are 1,600 hr. available to make the clothing.** **a. What are the variables?** *Number of shirts made and number of pajamas made.* **b. What are the constraints?** *How much time the manufacturer has and how much material is available.*

<table>
<tr>
<td>of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td>
<td>strategically.

MP.6 Attend to precision.</td>
<td>intersection of the corresponding half-planes.</td>
<td>c. Write inequalities for the constraints.

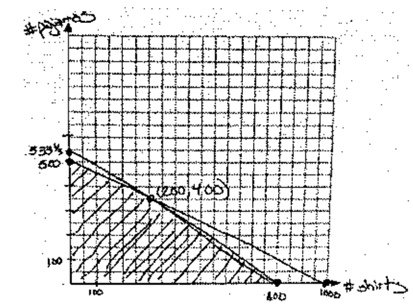
Let x = number of shirts, and let y = number of pajamas.

$x \geq 0$ and $y \geq 0$

$x + 2y \leq 1000$

$2x + 3y \leq 1600$

d. Graph the inequalities and shade the solution set.



e. What does the shaded region represent?

The various combinations of shirts and pajamas that it would be possible for the manufacturer to make.

f. The shaded region in a problem of this type is sometimes called the feasible region. Why does this name make sense?

This is the region that represents the number of shirts and pajamas that he can feasibly make given the constraints.</td>
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<tr>
<td>■ F.IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
■ F.IF.A.2. Use function notation, evaluate functions for inputs in their domains,</td>
<td>MP 2 Reason abstractly and quantitatively.

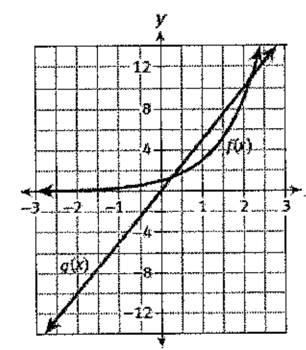
MP.6 Attend to precision.

MP.7 Look for and make use of structure.</td>
<td>Concept(s):
F(x) is an element in the range and x is an element in the domain.
Students are able to:
use the definition of a function to determine whether a relationship is a function.
use function notation once a relation is determined to be a function.
evaluate functions for given inputs in the domain.
explain statements involving function notation in the context of the problem.</td>
<td>Jerome is constructing a table of values that satisfies the definition of a function.
<table><tr><td>Input</td><td>−13</td><td>20</td><td>0</td><td>−4</td><td>11</td><td>−1</td><td>17</td><td></td></tr><tr><td>Output</td><td>−15</td><td>−11</td><td>−9</td><td>−2</td><td>−1</td><td>5</td><td>5</td><td>13</td></tr></table>What number(s) can be placed in the empty cell so that the table of values satisfies the definition of a function? Select all that apply.

A. −5 B. −1 C. 0 D. 2 E. 11 F. 17</td>
</tr>
</table>

and interpret statements that use function notation in terms of a context.		Learning Goal 3: Explain the definition of a function, including the relationship between the domain and range. Use function notation, evaluate functions and interpret statements in context.	
☐ F.LE.A.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. F.LE.A.1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. F.LE.A.1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. F.LE.A.1c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	MP.3 Construct viable arguments and critique the reasoning of others. MP.6 Attend to precision.	Concept(s): • Linear functions grow by equal differences over equal intervals. • Exponential functions grow by equal factors over equal intervals. Students are able to: • identify and describe situations in which one quantity changes at a constant rate. • identify and describe situations in which a quantity grows or decays by a constant percent. • show that linear functions grow by equal differences over equal intervals. • show that exponential functions grow by equal factors over equal intervals. Learning Goal 4: Distinguish between and explain situations modeled with linear functions and with exponential functions.	Determine whether the following ordered pairs represent a linear or an exponential relationship. An initial population of 5 squirrels increases by 9% each year for 10 years. Using x for years and y for the number of squirrels, write the equation that models this situation. How many squirrels will there be in 10 years? A car purchased for $34,000 is expected to lose value, or depreciate, at a rate of 6% per year. Using x for years and y for the value of the car, write the equation that models this situation. After how many years is the car first worth less than $21,500?
☐ F.LE.A.2. Construct linear and exponential functions - including arithmetic and geometric sequences - given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *[Algebra 1 limitation: exponential expressions with integer exponents] ■ F.IF.A.3. Recognize that sequences are functions,	MP 2 Reason abstractly and quantitatively. MP 4. Model with mathematics MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools	Concept(s): • Sequences are functions, sometimes defined and represented recursively. • Sequences are functions whose domain is a subset of integers. Students are able to: • create arithmetic and geometric sequences from verbal descriptions. • create arithmetic sequences from linear functions.	Write a geometric sequence. You must include the first four terms of your sequence. Identify your common ratio then write an equation to represent the rule. In (a)–(e), say whether the quantity is changing in a linear or exponential fashion. Write the equation of the function.

sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*	strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure.	• create geometric sequences from exponential functions. • identify recursively defined sequences as functions. • create linear and exponential functions given – a graph; – a description of a relationship; – a table of values. Learning Goal 5: Write linear and exponential functions given a graph, table of values, or written description; construct arithmetic and geometric sequences.	a. A savings account, which earns no interest, receives a deposit of $723 per month. b. The value of a machine depreciates by 17% per year. c. Every week, 9/10 of a radioactive substance remains from the beginning of the week. d. A liter of water evaporates from a swimming pool every day. e. Every 124 minutes, 1/2 of a drug dosage remains in the body.
☐ F.BF.A.1. Write a function that describes a relationship between two quantities. 1a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ■ A.SSE.A.1. Interpret expressions that represent a quantity in terms of its context A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients. A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* *[Algebra 1 limitation:	MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics	Concept(s): No new concept(s) introduced Students are able to: • given a context, write an explicit expressions, a recursive process or steps for calculation for linear and exponential relationships. • interpret parts of linear and exponential functions in context. Learning Goal 6: Write explicit expressions, recursive processes and steps for calculation from a context that describes a linear or exponential relationship between two quantities.	All exponential functions are in the form $y = a(b)^x$. What values of b make it an exponential growth function? What values of b make it an exponential decay function? If you have $200 to invest for 10 years, would you rather invest your money in a bank that pays 7% simple interest or in a bank that pays 5% interest compounded annually? Is there anything you could change in the problem that would make you change your answer? Show how you arrived at your answer using a recursive formula and then using an explicit formula.

exponential expressions with integer exponents]			
☐ A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* *[Algebra 1: limit to exponential expressions with integer exponents]	MP.1 Make sense of problems and persevere in solving them. MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure	Concept(s): No new concept(s) introduced Students are able to: • use the properties of exponents to simplify or expand exponential expressions, recognizing these are equivalent forms. Learning Goal 7: Use properties of exponents to produce equivalent forms of exponential expressions in one variable.	Simplify. $(a^{-2}b^3)^{-2}\,(b^3c^{-4})^2$ $\left(\frac{3}{x^2}\right)^3$ $\left(\frac{1}{3a^3b^2}\right)^{-4} \cdot (-3a^{10}b^9)^{-1}$
■ F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *[Focus on exponential functions] ☐ F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.	MP 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • given a verbal description of a relationship, sketch linear and exponential functions. • identify intercepts and intervals where the function is positive/negative. • interpret parameters in context. • determine the *practical* domain of a function. Learning Goal 8: Sketch graphs of linear and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context.	Examine the graphs of $f(x) = 3^x$ and $g(x) = 5x$, shown below.  a. Estimate the values of x for which $f(x)$ is greater than $g(x)$. b. Estimate the values of x for which $g(x)$ is greater than $f(x)$.

<table>
<tr>
<td>■ F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>☐ F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
*[Limit to linear and exponential]
■ F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

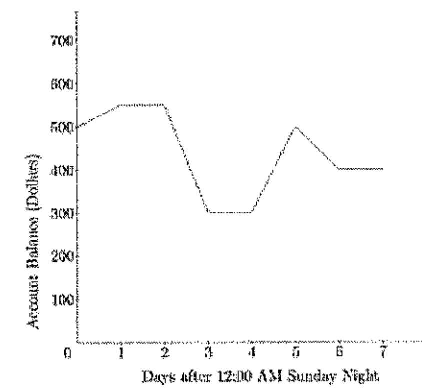
MP.8 Look for and express regularity in repeated reasoning.</td>
<td>Concept(s):
• Rate of change of non-linear functions varies.
Students are able to:
• compare key features of two linear functions represented in different ways.
• compare key features of two exponential functions represented in different ways.
• calculate the rate of change from a table of values or from a function presented symbolically.
• estimate the rate of change from a graph.

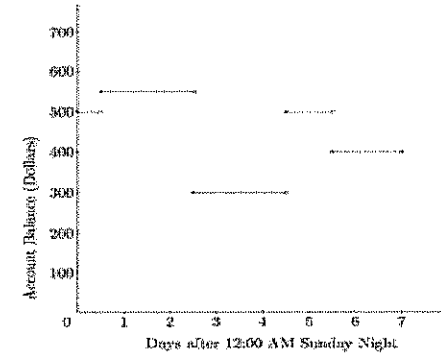
Learning Goal 9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Learning Goal 10: Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph.</td>
<td>In 2007, Zack bought a new car for $17,500. The table below shows the value of the car between 2007 and 2012.
<table><tr><th>Year</th><th>Car Value (in dollars)</th></tr><tr><td>2007</td><td>17,500</td></tr><tr><td>2008</td><td>12,767</td></tr><tr><td>2009</td><td>11,394</td></tr><tr><td>2010</td><td>10,091</td></tr><tr><td>2011</td><td>8,881</td></tr><tr><td>2012</td><td>7,857</td></tr></table>
Part A. Calculate the average rate of change of the value of the car between 2007 and 2008. Explain what your answer means in terms of the car's value over this interval.

Part B. Calculate the average rate of change of the value of the car between 2008 and 2012. Explain what your answer means in terms of the car's value over this interval.

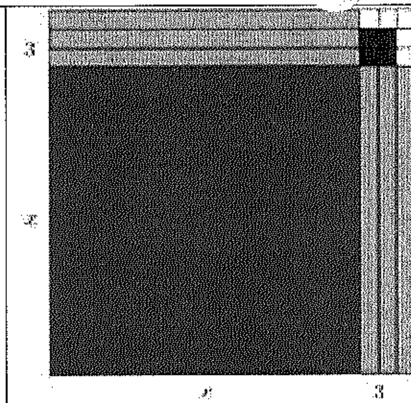
Part C. Compare the values from Part A and Part B. What can you conclude based on this comparison along with the data in the table in terms of the car's value over the time period</td>
</tr>
</table>

			shown in the table? Use words, numbers and/or pictures to show your work.
☐ F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7b. Graph piecewise-defined functions, including step functions and absolute value functions.	MP.1 Make sense of problems and persevere in solving them. MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	Concept(s): • Piecewise-defined functions may contain discontinuities. • Absolute value functions are piecewise functions. Students are able to: • graph linear, piecewise-defined functions. • graph more complicated cases of functions using technology. • identify and describe key features of the graphs of piecewise-defined functions . Learning Goal 11: Graph linear and piecewise-defined functions (including step and absolute value functions) expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph.	Graph f(x) = $\|x\|$ and the resulting graphs of the expanded functions. Compare and contrast the behavior of these graphs to f(x) = x and its expanded versions f(x) = ax + c. At the beginning of the week, Jessie had $500 in her bank account. She deposited a check for $50 on Tuesday and then paid $250 in rent on Wednesday. On Friday, Jessie deposited $200 in the account and then on Saturday she paid $50 for groceries from her bank account. Jessie made the following graph for the balance in her bank account during this week:  a. Is the depiction of how the account balance varies over the week accurate? Explain. b. How can Jessie graphically represent the bank account balance in a way that better shows how it changes?

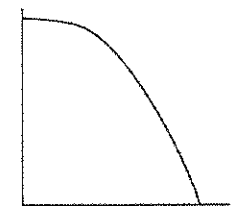
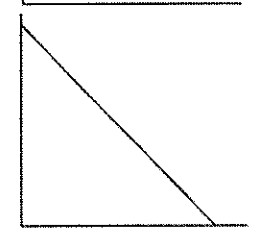


Unit 2 Vocabulary
System, substitution, elimination, linear combination, consistent and inconsistent system, dependent and independent system, infinitely many solutions, no solution, half-plane, exponent, negative exponent power, base, order of magnitude, Power of a Product/Quotient Property, Product/Quotient of a Power Property, Power of a Power Property, reciprocal, scientific notation, exponential function, exponential growth, exponential decay, compound interest, initial amount, growth/decay factor, growth/decay rate, time.

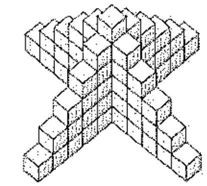
Unit 3 Algebra I			
Content & Practice Standards	**Standards for Mathematical Practice**	**Critical Knowledge & Skills**	**Examples**
A.APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	Concept(s): • Polynomials form a system analogous to the integers. • Polynomials are closed under the operations of addition, subtraction, and multiplication. Students are able to: • Add and subtract polynomials. • Multiply polynomials. • Recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences. • Recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences. Learning Goal 1: Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable.	A.APR.A.1 Powers of 11 Felicia notices what appears to be an interesting pattern between powers of 11 and powers of $x + 1$: $11^0 = 1$ $(x+1)^0 = 1$ $11^1 = 11$ $(x+1)^1 = x+1$ $11^2 = 121$ $(x+1)^2 = x^2 + 2x + 1$ *The digits of the number 11^n are the same as the coefficients of th polynomial $(x+1)^n$. Is this always true?* a. Does this pattern continue for $n = 3$ and $n = 4$? b. What is the answer to Felicia's question? A.SSE.A.2 Equivalent Expressions Find a value for *a*, a value for *k*, and a value for *n* so that $(3x+2)(2x-5)=ax^2+kx+n$.
A.REI.B.4. Solve quadratic equations in one variable. A.REI.B.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A.REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the	MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): • Multiple methods for solving quadratic equations. • Transforming a quadratic equation into the form $(x - p)^2 = q$ yields an equation having the same solutions. Students are able to: • Use the method of completing the square to transform a quadratic equation in x into an equation of the form $(x - p)^2 = q$. • Derive the quadratic formula from $(x - p)^2 = q$. • Solve a quadratic equation in one variable by inspection. • Solve quadratic equations in one variable by taking square roots. • Solve a quadratic equation in one variable by	A.REI.B.4 Visualizing Completing the Square Enrico has discovered a geometric technique for "completing the square" to find the solutions of quadratic equations. To solve the equation $x2+6x+4=0$, Enrico draws a square of dimension x by x and attaches 6 strips (3 of dimension 1 by x and 3 of dimension x by 1) to make the picture below:

square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.		completing the square. • Solve a quadratic equation in one variable using the quadratic formula. • Solve a quadratic equation in one variable by factoring. • Strategically select, as appropriate to the initial form of the equation, a method for solving a quadratic equation in one variable. • Write complex solutions of the quadratic formula in $a \pm bi$ form. • Analyze the quadratic formula, recognizing the conditions leading to complex solutions (discriminant). Learning Goal 2: Derive the quadratic formula by completing the square and recognize when there are no real solutions. Learning Goal 3: Solve quadratic equations in one variable using a variety of methods (including inspection, taking square roots, factoring, completing the square, and the quadratic formula) and write complex solutions in $a \pm bi$ form.	 To complete the larger square, Enrico adds 9 squares of dimension 1 by 1. He has 4 of them in his initial expression so he needs five more as shown in the picture. So the picture represents the equation $(x+3)^2=(x^2+6x+4)+5$ 1. Explain how Enrico's method helps find the roots of $x^2+6x+4=0$. 2. Help Enrico draw a picture for, and then solve, the equation $x^2-6x+4=0$. <u>A.REI.B.4 Braking Distance</u> The braking distance, in feet, of a car traveling at v miles per hour is given by $d=2.2v+v220$. 1. What is the braking distance, in feet, if the car is going 30 mph? 60 mph? 90 mph? 2. Suppose that the car took 500 feet to brake. Use your computations in part (a) to make a prediction about how fast it was going wher the brakes were applied. 3. Use a graph of the distance equation to determine more precisely how fast it was going when the brakes were applied, and check your answer using the quadratic

			formula. <u>A.REI.B.4 Two Squares are Equal</u> Solve the quadratic equation $x^2=(2x-9)^2$ using as many different methods as possible.
■ A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.	MP 2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	Concept(s): No new concept(s) introduced Students are able to: • Create quadratic equations in one variable. • Use quadratic equations to solve real world problems. Learning Goal 4: Create quadratic equations in one variable and use them to solve problems.	<u>A-CED.A.1 Paying the rent</u> A checking account is set up with an initial balance of $4800, and $400 is removed from the account each month for rent (no other transactions occur on the account). 1. Write an equation whose solution is the number of months, *m*, it takes for the account balance to reach $2000. 2. Make a plot of the balance after *m* months for *m*=1,3,5,7,9,11 and indicate on the plot the solution to your equation in part (a).
■ F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ■ F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it	MP.4 Model with mathematics. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • Interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem. • Sketch graphs of quadratic functions given a verbal description of the relationship between the quantities. • identify intercepts and intervals where function is increasing/decreasing • Determine the practical domain of a function. Learning Goal 5: Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its	<u>F.IF.B.4 Words – Tables - Graphs</u> Below are 4 verbal descriptions, 3 graphs, and 3 tables of values. Match each of the following descriptions with an appropriate graph and table of values. Create the missing graph and the missing table of values. 1. The weight of your jumbo box of cereal decreases by an equal amount every week. 2. The value of the bicycle depreciated rapidly at first, but its value declined more slowly as time went on. 3. The tennis ball is dropped off the roof of a skyscraper. 4. For a while it looked like the decline in profits was slowing down, but then they began

describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*		graph.	declining more rapidly. A. x 0 1 2 3 4 5 y 400 384 336 256 144 0 B. x 0 1 2 3 4 5 y 400 184 98 63 49 43 C. x 0 1 2 3 4 5 y 400 253 218 216 181 34 1.  2.  3.  <u>F.IF.B.5 The restaurant</u> A restaurant is open from 2 pm to 2 am on a

			certain day, and a maximum of 200 clients can fit inside. If $f(t)$ is the number of clients in the restaurant t hours after 2 pm that day, 1. What is a reasonable domain for f? 2. What is a reasonable range for f?
☐ A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.B.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.7 Look for and make use of structure.	Concept(s): • Alternate, equivalent forms of a quadratic expression may reveal specific attributes of the function that it defines. Students are able to: • Factor a quadratic expression for the purpose of revealing the zeros of a function. • Complete the square for the purpose of revealing the maximum or minimum of a function. Learning Goal 6: Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlights particular properties such as the zeros or the maximum or minimum value of the function.	A.SSE.B.3 Profit of a company The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If p is the price of the item, then three equivalent forms for the profit are: Standard form: :$-2p^2+24p-54$ Factored form: $-2(p-3)(p-9)$ Vertex form: $-2(p-6)^2+18$. Which form is most useful for finding 1. The prices that give a profit of zero dollars? 2. The profit when the price is zero? 3. The price that gives the maximum profit? A.SSE.B.3 Rewriting a Quadratic Expression 1. What is the minimum value taken by the expression $(x-4)^2+6$? How does the structure of the expression help to see why? 2. Rewrite the quadratic expression x^2-6x-3 in the form $(x\_\_\_)^2$ - ___ and find its minimum value. 3. Rewrite the quadratic expression $-2x^2+4x+3$ in the form ___$(x - ___)^2 +$___. What is its maximum value? Explain how you know.
☐ F.BF.A.1. Write a function that describes a relationship	MP.2 Reason abstractly and	Concept(s): No new concept(s) introduced	F.BF.A.1a Skeleton Tower

between two quantities. F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.	quantitatively. MP.4 Model with mathematics.	Students are able to: • Given a context, write explicit expressions, a recursive process or steps for calculation for quadratic relationships. Learning Goal 7: Given a context, write an explicit expression, a recursive process or steps for calculation for quadratic relationships.	 1. How many cubes are needed to build this tower? 2. How many cubes are needed to build a tower like this, but 12 cubes high? Justify your reasoning. 3. How would you calculate the number of cubes needed for a tower n cubes high?
☐ F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *[emphasize quadratic functions] ☐ F.IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. F.IF.C.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. ■ F.IF.C.9. Compare properties of two functions each	MP.1 Make sense of problems and persevere in solving them. MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.8 Look for and express regularity in repeated reasoning.	Concept(s): No new concept(s) introduced Students are able to: • Graph quadratic functions expressed symbolically. • Graph more complicated cases of quadratic functions using technology. • Identify and describe key features of the graphs of quadratic functions. • Given two quadratic functions, each represented in a different way, compare the properties of the functions. Learning Goal 8: Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.	<u>F.IF.C.7a Graphs of Quadratic Functions</u> Graph these equations on your graphing calculator at the same time. What happens? Why? $y_1 = (x-3)(x+1)$ $y_2 = x^2-2x-3$ $y_3 = (x-1)^2-4$ $y_4 = x^2-2x+1$ • Below are the first three equations from the previous problem. $y_1 = (x-3)(x+1)$ $y_2 = x^2-2x-3$ $y_3 = (x-1)^2-4$ These three equations all describe the same function. What are the coordinates of the following points on the graph of the function? From which equation is each point most easily determined? Explain. 1. vertex: _____ 2. y-intercept: _____ 3. x-intercept(s): _____

represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

- Make up an equation for a quadratic function whose graph satisfies the given condition. Use whatever form is most convenient.

1. Has a vertex at (−2,−5).
2. Has a y-intercept at (0,6)
3. Has x-intercepts at (3,0) and (5,0)
4. Has x-intercepts at the origin and (4,0)
5. Goes through the points (4,2) and (1,2)

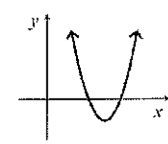
F.IF.C.8a Springboard Dive

- Suppose $h(t)=-5t^2+10t+3$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard.

1. How high above the water is the springboard? Explain how you know.
2. When does the diver hit the water?
3. At what time on the diver's descent toward the water is the diver again at the same height as the springboard?
4. When does the diver reach the peak of the dive?

F.IF.C.8a Which Function?

Which of the following could be the function of a real variable x wh graph is shown below? Explain.



$f_1(x) = (x+12)^2+4$ $\qquad$ $f_5(x) = -4(x+2)(x+3)$

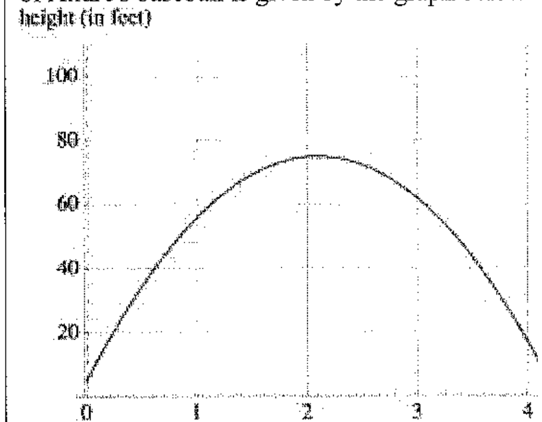
$f_2(x) = -(x-2)^2-1$ $\qquad$ $f_6(x) = (x+4)(x-6)$

$f_3(x) = (x+18)^2-40$ $\qquad$ $f_7(x) = (x-12)(-x+18)$

$f_4(x) = (x-12)^2-9$ $\qquad$ $f_8(x) = (24-x)(40-x)$

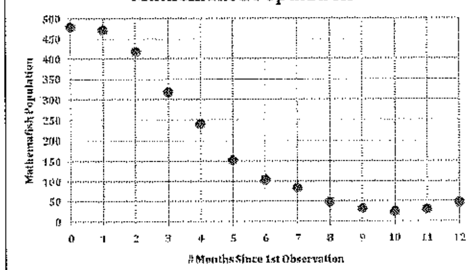
F.IF.C.9 Throwing Baseballs

Suppose Brett and Andre each throw a baseball

			into the air. The height of Brett's baseball is given by $$h(t) = -16t^2 + 79t + 6,$$ where h is in feet and t is in seconds. The height of Andre's baseball is given by the graph below:  Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher. 1. Who is right? 2. How long is each baseball airborne? 3. Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).
■ F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ☐ F.LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	Concept(s): • A quantity increasing exponentially eventually exceeds a quantity increasing quadratically. Students are able to: • Calculate the rate of change of a quadratic function from a table of values or from a function presented symbolically. • Estimate the rate of change from a graph of a quadratic function.	<u>F.IF.B.6 Mathemafish Population</u> You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

<table>
<tr>
<td>increasing linearly, quadratically, or (more generally) as a polynomial function.</td>
<td></td>
<td>• Analyze graphs and tables to compare rates of change of exponential and quadratic functions.

Learning Goal 9: Calculate and interpret the average rate of change of a quadratic function presented symbolically or as a table. Estimate and compare the rates of change from graphs of quadratic and exponential functions.</td>
<td># months since 1st measurement: 0 1 2 3 4 5 6 7 8 9 10 11 12
Mathemafish population: 480 472 417 318 240 152 103 84 47 32 24 29 46

Mathemafish Population


Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA's intervention plan in order to validate funding for your project.

What to include in your summary report:

• Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.
• When was the population decreasing the fastest?
• During what month did you notice the largest effects of the EPA intervention?
• Explain the overall effects of the intervention.
• Remember to justify all your conclusions using supporting evidence</td>
</tr>
<tr>
<td>◎ F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and</td>
<td>MP.3 Construct viable arguments and critique the reasoning of others.

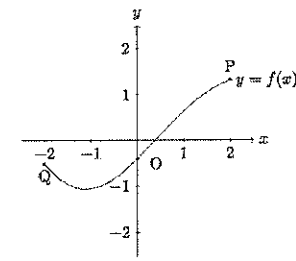
MP.5 Use appropriate tools strategically.

MP.7 Look for and make use</td>
<td>Concept(s):

• Characteristics of even and odd functions in graphs and algebraic expressions
• Vertical and horizontal shifts
Students are able to:

• Perform transformations on graphs of linear and</td>
<td><u>F.BF.B.3 Identifying Even and Odd Functions</u>

Determine whether each of these functions is odd, even, or neither. Use algebraic methods on all of the functions. You may start out by looking</td>
</tr>
</table>

illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	of structure.	quadratic functions. • Identify the effect on the graph of replacing f(x) by – $f(x)+k$; – $k\,f(x)$; – $f(kx)$; – *and* $f(x+k)$ for specific values of k (both positive and negative). • Identify the effect on the graph of combinations of transformations. • Given the graph, find the value of k. • Illustrate an explanation of the effects on linear and quadratic graphs using technology. • Recognize even and odd functions from their graphs and from algebraic expressions for them. Learning Goal 10: Identify the effects of transformations and combinations of transformations [$f(x)+k$, $k\,f(x)$, $f(kx)$, and $f(x+k)$] on a function; find the value of k given the graph.	at a graph, if you need to. 1. $f(x)=3^x+3^{-x}$ 2. $g(x)=2^x-2^{-x}$ 3. $h(x)=x^2+4x-2$ 4. $j(x)=x^3-4x$ <u>F.BF.B.3 Transforming the graph of a function</u> The figure shows the graph of a function f whose domain is the interval $-2\le x\le 2$.  1. In (i)–(iii), sketch the graph of the given function and compare with the graph of f. Explain what you see. 1. $g(x)=f(x)+2$ 2. $h(x)=-f(x)$ 3. $p(x)=f(x+2)$ 2. The points labelled Q,O,P on the graph of f have coordinates $Q=(-2,-0.509),O=(0,-0.4),P=(2,1.309)$ What are the coordinates of the points corresponding to P,O,Q on the graphs of g,h, and p?
☐ A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations y	MP.1 Make sense of problems and persevere in solving them.	Concept(s): No new concept(s) introduced Students are able to:	<u>A.REI.D.11 Introduction to Polynomials – College Fund</u> When Marcus started high school, his

= f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	MP.5 Use appropriate tools strategically.	• Approximate the solution(x) to a system of equations comprised of a linear and a quadratic function by using technology to graph the functions, by making a table of values and/or by finding successive approximations. Learning Goal 11: Find approximate solutions of f(x) = g(x), where f(x) is a linear function and g(x) is a quadratic function by making a table of values, using technology to graph and finding successive approximations.	grandmother opened a college savings account. On the first day of each school year she deposited money into the account: $1000 in his freshmen year, $600 in his sophomore year, $1100 in his junior year and $900 in his senior year. The account earns interest of r% at the end of each year. When Marcus starts college after four years, he gets the balance of the savings account plus an extra $500. 1. If r is the annual interest rate of the bank account, the at the end of the year the balance in the account is multiplied by a growth factor of $x=1+r$. Find an expression for the total amount of money Marcus receives from his grandmother as a function of this annual growth factor x. 2. Suppose that altogether he receives $4400 from his grandmother. Use appropriate technology to find the interest rate that the bank account earned. 3. How much total interest did the bank account earn over the four years? 4. Suppose the bank account had been opened when Marcus started Kindergarten. Describe how the expression for the amount of money at the start of college would change. Give an example of what it might look like.
☐ A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available]	MP.7 Look for and make use of structure.	Concept(s): • General shape(s) and end behavior of cubic functions Students are able to: • find the zeros of a polynomial (quadratic and cubic). • test domain intervals to determine where f(x) is greater than or less than zero. • use zeros of a function to sketch a graph. Learning Goal 12: Identify zeros of cubic functions when suitable factorizations are available and use the zeros to	A.APR.B.3 Graphing from Factors 1 Graph the functions given by the equations $y=(x-1)(x+2)(x-5)$ and $y=3(x-1)(x+2)(x-5)$ with a viewing window $-10\leq x\leq 10$ and $-100\leq y\leq 100$. 1. Describe any similarities you see between the two graphs, and explain how you can see those similarities in the given equations. 2. Write an equation for a function whose graph in the xy-plane has x-intercepts at -9, -6, 0, and 4. Graph your equation to

		construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor)	verify that it works.
◎ N.RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	MP.3 Construct viable arguments and critique the reasoning of others. MP.6 Attend to precision.	Concept(s): • The sum or product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. Students are able to: • Explain and justify conclusions regarding sums and products of two rational numbers.. • Explain and justify conclusions regarding the sum of a rational and irrational number. • Explain and justify conclusions regarding the product of a nonzero rational and irrational number. Learning Goal 13: Explain and justify conclusions about sums and products of rational and irrational numbers.	<u>N.RN.B.3 Operations with Rational and Irrational Numbers</u> Experiment with sums and products of two numbers from the following list to answer the questions that follow: $5, \frac{1}{2}, 0, \sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}}, \pi.$ Based on the above information, conjecture which of the statements is ALWAYS true, which is SOMETIMES true, and which is NEVER true? 1. The sum of a rational number and a rational number is rational. 2. The sum of a rational number and an irrational number is irrational. 3. The sum of an irrational number and an irrational number is irrational. 4. The product of a rational number and a rational number is rational. 5. The product of a rational number and an irrational number is irrational. 6. The product of an irrational number and an irrational number is irrational.

Unit 3 Vocabulary			
Common Ratio	Binomial	Axis of symmetry	Vertex
Compound Interest	Degree of a monomial	Completing the square	Zero-Product Property
Decay Factor	Degree of a polynomial	Discriminant	
Exponential Decay	Difference of squares	Maximum	
Exponential Function	Factor by grouping	Minimum	
Exponential Growth	Monomial	Parabola	

Geometric Sequence	Perfect-square trinomial	Quadratic Equation
Growth Factor	Polynomial	Quadratic Formula
Interest Period	Standard form of a polynomial	Quadratic Function
Scientific Notation	Trinomial	Quadratic Parent Function

<table>
<tr><th colspan="2">Unit 4 Algebra I</th><th colspan="2"></th></tr>
<tr><th>Content & Practice Standards</th><th>Standards for Mathematical Practice</th><th>Critical Knowledge & Skills</th><th>Examples</th></tr>
<tr>
<td>◎ S.ID.A.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.</td>
<td>Concept(s): No new concept(s) introduced

Students are able to:
represent data with dot plots on the real number line.
represent data with histograms on the real number line.
represent data with box plots on the real number line.
Learning Goal 1: Represent data with plots (dot plots, histograms, and box plots) on the real number line.</td>
<td>Histograms

Step 1: Using your roll of pennies, record the dates on each coin (a stem and leaf plot may be helpful).
Step 2: Create a box plot of your data.
Step 3: Create a histogram using decades.

Using the scores from a test, create a box plot showing the quartiles. Scores: 100, 83, 73, 84, 62, 93, 87, 92, 75, 76, 69, 83, 72, 70, 74, 78, 81, 80, 91, 70, 99</td>
</tr>
<tr>
<td>◎ S.ID.A.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
◎ S.ID.A.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.</td>
<td>Concept(s):
Appropriate use of a statistic depends on the shape of the data distribution.
Standard deviation
Students are able to:
represent two or more data sets with plots and use appropriate statistics to compare their center and spread.
interpret differences in shape, center, and spread in context.
explain possible effects of extreme data points (outliers) when summarizing data and interpreting shape, center and spread.
Learning Goal 2: Compare center and spread of two or more data sets, interpreting differences in shape, center, and spread in the context of the data, taking into account the effects of outliers.</td>
<td>Compare the daily sales figures from two Bagel Barn locations by creating two box plots. What differences are there? What conclusions can you draw from the data?

<table>
<tr><th>Day</th><th>Kean U.</th><th>Morris Ave.</th></tr>
<tr><td>1</td><td>501</td><td>756</td></tr>
<tr><td>2</td><td>432</td><td>866</td></tr>
<tr><td>3</td><td>670</td><td>776</td></tr>
<tr><td>4</td><td>832</td><td>972</td></tr>
<tr><td>5</td><td>165</td><td>1024</td></tr>
<tr><td>6</td><td>699</td><td>735</td></tr>
<tr><td>7</td><td>423</td><td>821</td></tr>
<tr><td>8</td><td>532</td><td>699</td></tr>
<tr><td>9</td><td>622</td><td>842</td></tr>
<tr><td>10</td><td>647</td><td>933</td></tr>
<tr><td>11</td><td>245</td><td>1013</td></tr>
<tr><td>12</td><td>156</td><td>987</td></tr>
<tr><td>13</td><td>639</td><td>836</td></tr>
<tr><td>14</td><td>723</td><td>921</td></tr>
</table></td>
</tr>
</table>

<table>
<tr>
<td>☐ S.ID.B.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP.5 Use appropriate tools strategically.

MP.7 Look for and make use of structure.</td>
<td>Concept(s):
• Categorical variables represent types of data which may be divided into groups.
Students are able to:
• construct two-way frequency tables for categorical data.
• interpret joint, marginal and conditional relative frequencies in context.
• explain possible associations between categorical data in two-way tables.
• identify and describe trends in the data.

Learning Goal 3: Summarize and interpret categorical data for two categories in two-way frequency tables; explain possible associations and trends in the data.</td>
<td>Given the data below construct a joint frequency table. Find the marginal frequencies and the relative frequencies.

Morning observation of vehicle color:
<table><tr><td>Silver</td><td>123</td></tr><tr><td>White</td><td>57</td></tr><tr><td>Blue</td><td>74</td></tr><tr><td>Green</td><td>20</td></tr><tr><td>Red</td><td>6</td></tr><tr><td>Yellow</td><td>7</td></tr><tr><td>Black</td><td>83</td></tr><tr><td>Multiple</td><td>20</td></tr><tr><td>Other</td><td>15</td></tr></table>
Evening observation of vehicle color:
<table><tr><td>Silver</td><td>157</td></tr><tr><td>White</td><td>62</td></tr><tr><td>Blue</td><td>65</td></tr><tr><td>Green</td><td>16</td></tr><tr><td>Red</td><td>24</td></tr><tr><td>Yellow</td><td>12</td></tr><tr><td>Black</td><td>55</td></tr><tr><td>Multiple</td><td>32</td></tr><tr><td>Other</td><td>29</td></tr></table>
What do you see when comparing the morning and evening? How does the data change?</td>
</tr>
<tr>
<td>☐ S.ID.B.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

☐ S.ID.B.6a. Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.</td>
<td>MP.1 Make sense of problems and persevere in solving them.

MP 2 Reason abstractly and quantitatively.
MP.4 Model with mathematics.

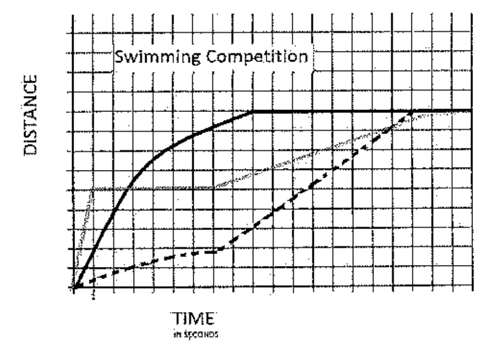
MP.5 Use appropriate tools strategically.</td>
<td>Concept(s): No new concept(s) introduced

Students are able to:
• fit a function to data using technology.
• solve problems using functions fitted to data (prediction equations).
• interpret the intercepts of models in context.
• plot residuals of linear and non-linear functions.</td>
<td>Create scatter plots from the data below and use each to create a function to fit the data.

A: Time and height of a ball:

(0,4); (1,9); (2,12); (3,13); (4, 12)

What type of graph would best fit the data?</td>
</tr>
</table>

Emphasize linear, quadratic, and exponential models. ☐ S.ID.B.6b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.	MP.6 Attend to precision.	• analyze residuals in order to informally evaluate the fit of linear and non-linear functions. Learning Goal 4: Fit functions to data using technology, plot residuals and informally assess the fit of linear and non-linear functions by analyzing residuals.	What is the maximum height of the ball? What do the zeroes of the graph represent? What would you predict the value of the function to be when x = 6 seconds? B: Height of a candle in inches over time: (0, 6); (2, 5); (3, 4.5); (1, 5.5); (0.5, 5.75); (2.5, 4.75) What type of graph would best fit the data? What do the x and y intercepts represent? What would you predict the value of the function to be when x = 4 hours?
■ F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ■ F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	MP.4 Model with mathematics. MP.6 Attend to precision.	Concept(s): No new concept(s) introduced Students are able to: • interpret maximum/minimum and intercepts of functions from graphs and tables in the context of the problem. • sketch graphs of functions given a verbal description of the relationship between the quantities. • identify intercepts and intervals where function is increasing/decreasing. • determine the practical domain of a function . Learning Goal 5: Interpret key features of functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a function, showing key features and relating the domain of the function to its graph.	 The graph shows the distance each swimmer travelled over the time in seconds. Write a news report about the race. Describe what happened with each swimmer, who won, and any unusual events as shown on the graphs.

Unit 4 Vocabulary
outliers, mean, median, mode, central tendencies, scatter plot, box plot, dot plot, histogram, frequency table, joint frequency, conditional frequency, conditional relative frequency, intercepts, intervals, relative maximum, relative minimum, models, residuals, quartile, standard deviation, mean absolute deviation, interquartile range

Research-Based Effective Teaching Strategies	**21st Century Learning Skills**
Task/Activities that solidifies mathematical concepts Use questioning techniques to facilitate learning Reinforcing Effort, Providing Recognition Practice , reinforce and connect to other ideas within mathematics Promotes linguistic and nonlinguistic representations Cooperative Learning Setting Objectives, Providing Feedback Varied opportunities for students to communicate mathematically Use technological and /or physical tools	Teamwork and Collaboration Initiative and Leadership Curiosity and Imagination Innovation and Creativity Critical thinking and Problem Solving Flexibility and Adaptability Effective Oral and Written Communication Accessing and Analyzing Information

Formative Assessment	**Summative Assessment**	**Technology**
Short constructed responses Extended responses Checks for understanding Exit tickets Teacher observation Projects Timed Practice Test – Multiple Choice &	Data Analysis Project End of Unit Assessment	NJ CORE Annenberg Learning : Insight into Algebra 1 Mathematics Assessment Projects Get the Math Achieve the Core

Open-Ended Questions		Webmath.com sosmath.com Mathplanet.com Interactive Mathematics.com Illustrative Mathematics Inside Mathmatics.org Asia Pacific Economic Cooperation : Lesson Study Videos Genderchip.org Interactive Geometry Mathematical Association of America National Council of Teachers of Mathematics Desmos.com Shodor.org