TOWNSHIP OF UNION PUBLIC SCHOOLS

Advanced Math
Adopted June 17, 2015
Updated December 18, 2018

District Mission Statement

The Township of Union Board of Education believes that every child is entitled to an education, designed to meet his or her individual needs, in an environment that is conducive to learning. State standards, federal and state mandates, and local goals and objectives, along with community input, must be reviewed and evaluated on a regular basis to ensure that an atmosphere of learning is both encouraged and implemented. Furthermore, any disruption to or interference with a healthy and safe educational environment must be addressed, corrected, or, when necessary, removed, in order for the district to maintain the appropriate educational setting.

District Philosophy Statement

The Township of Union Public School District, as a societal agency, reflects democratic ideals and concepts through its educational practices. It is the belief of the Board of Education that a primary function of the Township of Union Public School System is the formulation of a learning climate conducive to the needs of all students in general, providing therein for individual differences. The school operates as a partner with the home and community.

Course Description

The purpose of this course is to provide students with a working knowledge of trigonometry, a sampling of discrete mathematics topics, and an introduction to calculus topics to prepare them for future courses in mathematics.

Students will apply their reasoning abilities when recognizing patterns, making generalizations, and drawing logical conclusions. Students will use these skills in other disciplines and in real-life situations. They will use technology to evaluate and validate solutions.

Recommended Textbooks:

Precalculus: Graphical, Numerical, Algebraic
Authors: Demana, Waits, Foley and Kennedy

Curriculum Units

Unit 1: Polynomial Functions
Unit 2: Angles and Trigonometric Functions
Unit 3: Applications of Trigonometric Functions
Unit 4: Matrices
Unit 5: Vectors and Polar Coordinates
Unit 6: Conic Sections
Unit 7: Differentiation and Integration

Pacing Guide

Content	Number of Days
Unit 1: POLYNOMIAL FUNCTIONS	45
Unit 2: ANGLES AND TRIGONOMETRIC FUNCTIONS	45
Unit 3: APPLICATIONS OF TRIGONOMETRIC FUNCTIONS	15
Unit 4: MATRIX ALGEBRA	15
Unit 5: VECTORS AND POLAR COORDINATES	15
Unit 6: CONIC SECTIONS	15
Unit 7: DIFFERENTIATION AND INTEGRATION	10

Unit 1 (ANALYZING POLYONOMIAL FUNCTIONS)

NJSLS	NJSLS Content	Learning Activities
A-APR-1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
A-APR-2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
A-APR-3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
A-APR-4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x2 + y2)2 = (x2 - y2)2 + (2xy)2$ *can be used to generate Pythagorean triples.*	
A-APR-5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)$n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	
F-IF-1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
F-IF-2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
F-IF-3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	

F-IF-4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	
F-IF-5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	
F-IF-6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	
F-IF-7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	

Unit 1 Proficiencies

Students will be able to:

- ANALYZE POLYNOMIAL FUNCTIONS
- FIND THE INVERSE OF POLYNOMIAL FUNCTIONS

- DETERMINE IF THE INVERSE IS A FUNCTION?

Suggested Differentiation for Unit 1

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

Curriculum Resources

- Textbook - Precalculus: Graphical, Numerical, Algebraic
- Sullivan Pre-Calculus Playlist https://www.youtube.com/user/SullivanPrecalc9e/playlists
- Connect the Dots; https://www.kqed.org/mindshift/25063/connecting-the-dots-teaching-how-to-think

Formative Assessments	**Summative Assessments**
Homework Classroom whiteboard problem solving	Quiz Chapter Test

Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Projects

Interdisciplinary Connections/Technology
Physics: Rates of change can be applied to motion, electricity, heat, light, and astronomy.

Unit 2 ANGLES AND TRIGONOMETRIC FUNCTIONS

NJSLS	NJSLS Content	Learning Activities

F-TF-1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
F-TF-2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	
F-TF-3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.	
F-TF-4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	
F-TF-5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	
F-TF-6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	
F-TF-7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★	

F-TF-8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
F-TF-9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Unit 2 Proficiencies

Students will be able to:

- USE TRIGONOMETRIC FUNCTONS AND THEIR RELATION TO THE UNIT CIRCLE, RIGHT TRIANGLES AND ANGLES

Suggested Differentiation for Unit 2

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
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- **Tier 3 Learners:**
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Polar Coordinates

- Converting Polar Coordinates (YouTube)
- Converting Polar Equations to Rectangular Equations (YouTube)
- Converting Rectangular Equations to Polar Equations (YouTube)

Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects FIND THE SIX TRIGONOMETRIC VALUES OF AN ANGLE MEASURING 135° FIND THE AREA OF A TRIANGLE WITH TWO SIDES MEASURING 7.5 CM AND 9 CM. AND AN INCLUDED ANGLE OF 100°

Interdisciplinary Connections/Technology

Physics: Rates of change can be applied to motion, electricity, heat, light, and astronomy.

Unit 3 (APPLICATIONS OF TRIGONOMETRIC FUNCTIONS)

NJSLS	NJSLS Content	Learning Activities
A-APR-1	Understand that polynomials form a system analogous to the integers,	

	namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
A-APR-2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	
A-APR-3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	
A-APR-4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x2 + y2)2 = (x2 - y2)2 +$ $(2xy)2$ *can be used to generate Pythagorean triples.*	
A-APR-5	(+) Know and apply the Binomial Theorem for the expansion of $(x$ $+ y)$n in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1	

Unit 3 Proficiencies

Students will be able to:

- STUDENT WILL USE KNOWLEDGE OF TRIANGLE TRIGONOMETRY TO SOLVE PROBLEMS DONE BY PILOTS, SURVEYORS AND NAVIGATORS.

Suggested Differentiation for Unit 3

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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- Converting Rectangular Equations to Polar Equations (YouTube)

Formative Assessments	Summative Assessments

Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects A SHORELINE RUNS NORTH-SOUTH, AND A BOAT IS DUE EAST OF THE SHORELINE. THE BEARINGS OF THE BOAT FROM TWO POINTS ON THE SHORE ARE 110° AND 100°. ASSUME THE TWO POINTS ARE 550 FT APART. HOW FAR IS THE BOAT FROM THE SHORE? A POINT ON THE TIP OF A TUNING FORK VIBRATES IN HARMONIC MOTION DESCRIBED BY THE EQUATION d=14 sin(wt). FIND w FOR A TUNING FORK THAT HAS A FREQUENCY OF 528 VIBRATION PER SEC.?
Interdisciplinary Connections/Technology	
Aeronautics: Triangle Trig to solve problems done by pilots, surveyors, and navigators.	

Unit 4 (VECTORS)

NJSLS	NJSLS Content	Learning Activities
N-VM-1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $\|v\|$, $\|\|v\|\|$, v).	
N-VM-2	(+) Find the components of a vector by subtracting the	

	coordinates of an initial point from the coordinates of a terminal point.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
N-VM-3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.	
N-VM-4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	
N-VM-5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$. b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|c\boldsymbol{v}\| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	

Unit 4 Proficiencies
Students will be able to: • To use vectors to solve force problems

Unit 5 (MATRICES)

NJSLS	NJSLS Content	Learning Activities
N-VM-6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	
N-VM-7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	
N-VM-8	(+) Add, subtract, and multiply matrices of appropriate dimensions.	
N-VM-9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive	

	properties.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities Flipped classroom
N-VM-10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	
N-VM-11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	
N-VM-12	(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.	

Unit 5 Proficiencies

Students will be able to:

- **TO USE MATRICES TO ORGANIZE, MANIPULATE AND DISPLAY INFORMATION**

Suggested Differentiation for Unit 5

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.

- USE POLAR COORDINATES TO SOLVE PROBLEMS IN 3 DIMENSIONAL SPACE.

Suggested Differentiation for Unit 4

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
  - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

Curriculum Resources

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Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects FIND THE COMPONENT FORM AND MAGNITUDE OF THE VECTOR PQ IF P= (-2,2) AND Q=(3,4) FIND THE DOT PRODUCT OF U AND V IF U=<5,3> AND U=<12,4> FIND |U| IF U=<2,-3> THE LOCATIONS OF TWO SHIPS FROM MAYS LANDING LIGHTHOUSE, GIVEN IN POLAR COORDINATES, ARE (3 MI, 170°) AND (5 MIN, 150°). FIND THE DISTANCE BETWEEN THE SHIPS

Interdisciplinary Connections/Technology

Physics: To use vectors to solve force problems

Astronomy: USE POLAR COORDINATES TO SOLVE PROBLEMS IN 3 DIMENSIONAL SPACE.

- Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.

- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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Formative Assessments	**Summative Assessments**
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quiz Chapter Test Projects **SOLVE FOR X IF** **3X + A = B , WHERE** **A = |1| AND B = [4]**

	[3] [2]

Unit 6 (CONIC SECTIONS)

Unit 6 Proficiencies
Students will be able to: • Recognize a conic section. • Determine the standard form of a circle, ellipse, hyperbola, and parabola. • Identify the properties of circles, ellipses, hyperbolas, and parabolas. • Graph an equation to see the relationships between real-life quantities.
Suggested Differentiation for Unit 6
• **Tier 1 Learners:** ○ Have guided notes filled out at different levels according to ability. ○ Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.

- Group students by similar interest when working on application problems.
- Use mini lessons to reteach to those having difficulty.
- Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
- Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the assignment rather than completing the entire assignment.
- Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.

- **Tier 2 Learners:**
  - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
  - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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Formative Assessments	Summative Assessments
Homework Classroom whiteboard problem solving Exit tickets	Quiz Chapter Test Projects

Review Games Teacher Observations Use of technology (Google Suite) Do nows Oral questioning Short constructed responses	Quizzes on geometry of conics Quiz on Polar Equations of conics Test on Conics

Unit 7 (DIFFERENTIATION AND INTEGRATION)

NJSLS	NJSLS Content	Learning Activities
12.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	Homework review Direct instruction (board notes/presentations) Guided and independent practice Investigation activities
12.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	
12.F-IF.B.5	Relate the domain of a function to its graph and, where	

	applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function	Flipped classroom
12.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.	
12.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
12.F-LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.	

Unit 7 Proficiencies

Students will be able to:

- HOW DIFFERENTIAL AND INTEGRAL CALCULUS HELP US MAKE DECISIONS?

Suggested Differentiation for Unit 7

- **Tier 1 Learners:**
  - Have guided notes filled out at different levels according to ability.
  - Give assignments that contain tasks of varying difficulty. Each task should focus on essential learning that all students should master, but the tasks will vary in difficulty.
  - Group students by similar interest when working on application problems.
  - Use mini lessons to reteach to those having difficulty.
  - Group students so that each group contains all level learners. The tier 3 learners can serve as peer helpers.
  - Assign a basic homework assignment. Require students to spend a set amount of time to work (showing effort) on the

assignment rather than completing the entire assignment.
    - Allow students to choose a method for completing a project: video, PowerPoint, paper, or presentation.
- **Tier 2 Learners:**
    - Utilize foldables creating tangible products to help students digest information while incorporating several of the multiple intelligences.
- **Tier 3 Learners:**
    - Have problems posted around the room. Have students loop to specific questions based on difficulty.

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Formative Assessments	**Summative Assessments**
Homework Classroom whiteboard problem solving Exit tickets Review Games Teacher Observations Use of technology (Google Suite) Do nows	Quiz Chapter Test Projects **FIND THE DERIVATIVE OF THE FUNCTION**

Oral questioning Short constructed responses	$F(X)=3X^2 + 2X - 1$ **FIND THE SLOPE OF THE GRAPH AT THE INDICATED POINT** $F(X) = 2X^2$ **AT** $X = -1$ **FIND THE DEFINITE INTEGRAL BY COMPUTING AN AREA OF 5 dx FROM 3 TO 7**

Interdisciplinary Connections/Technology

Economics: Calculus can reduce production costs and optimize profits.
Epidemiology: The rate at which a disease spreads and how far it spreads can be modeled and analyzed using calculus.
Medicine: Medicine dosage rates can be modeled using differential equation.

Additional Suggested Modifications for Units

Below is an additional list of modifications and accommodations opportunities. This includes, but is not limited to,:

1. English Language Learners.
    a. Read written instructions.
    b. Model and provide examples
    c. Extended time on assessments when needed.
    d. Establish a non-verbal cue to redirect student when not on task.
    e. Students may use a bilingual dictionary.

English Language Development Standard 3: Language of Mathematics: English language learners communicate information, ideas and concepts necessary for academic success in the content area of mathematics.

2. Special Education/504 Students.
    a. Extended time on assessments when needed.
    b. Preferred seating to be determined by student and teacher.

c. Provide modified assessments when necessary.
d. Student may complete assessments in alternate setting when requested.
e. Establish a non-verbal cue to redirect student when not on task.
f. Maintain strong teacher / parent communication.
g. Conversion chart

New Jersey Student Learning Standards - Technology

8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

A. Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations

B. Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.

C. Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning.

E: Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.

F: Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

***See Guide for Technology Integration.**

Career Readiness Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.

- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

NJSLS 9.2 - Career Awareness, Exploration, and Preparation

9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career.